Updated October 2017



Small Steps Guidance and Examples

Block 2: Four Operations



Year 6 Autumn Term Small Steps Progression

Overview Small Steps

Add and subtract whole numbers
Multiply up to a 4-digit by 1-digit number
Short division
Division using factors
Long division (1)
Long division (2)
Long division (3)
Long division (4)
Common factors
Common multiples
Primes
Squares and cubes
Order of operations
Mental calculations and estimation
Reasoning from known facts

NC Objectives

Solve addition and subtraction multi step problems in contexts, deciding which operations and methods to use and why.

Multiply multi-digit number up to 4 digits by a 2-digit number using the formal written method of long multiplication.

Divide numbers up to 4 digits by a 2-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding as appropriate for the context.

Divide numbers up to 4 digits by a 2-digit number using the formal written method of short division, interpreting remainders according to the context.

Perform mental calculations, including with mixed operations and large numbers.

Identify common factors, common multiples and prime numbers.

Use their knowledge of the order of operations to carry out calculations involving the four operations.

Solve problems involving addition, subtraction, multiplication and division.

Use estimation to check answers to calculations and determine in the context of a problem, an appropriate degree of accuracy.

Add & Subtract Integers

Notes and Guidance

Children consolidate their knowledge of column addition and subtraction.

They use these skills to solve multi step problems in a range of contexts.

Mathematical Talk

- What happens when there is more than 10 in a place value column?
- Can you make an exchange between columns? How can we find the missing digits? Can we use the inverse?
- Is column method always the best method?
- When should we use our mental methods?

Varied Fluency

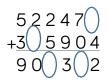
1	Calculate	
	34621 + 25734	4761325 938052
	 67,832 + 5,258 = 834,501 - 193,642	2=



A four-bedroom house costs £450,000 A three-bedroom house costs £199,000 less. How much does the three-bedroom house cost? What method did you use to find the answer?

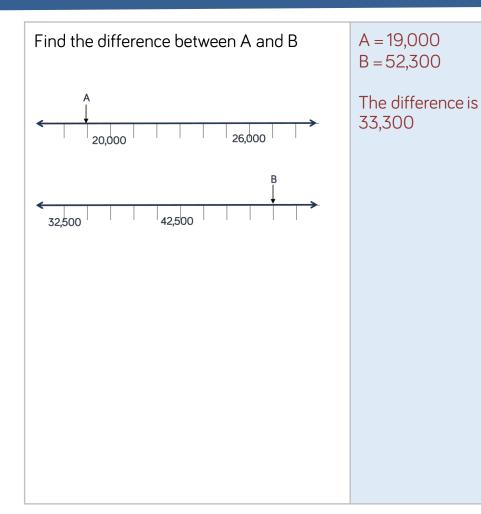


All the missing digits are the same. Find the missing digits



Add & Subtract Integers

Reasoning and Problem Solving



Here is a bar model. B 631,255

A is an odd number which rounds to 100,000 to the nearest ten thousand. It has a digit total of 30

B is an even number which rounds to 500,000 to the nearest hundred thousand. It has a digit total of 10

A and B are both multiples of 5 but end in different digits.

Possible answer:

99,255 + 532,000 = 631,255

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Multiply 4-digits by 2-digits

Notes and Guidance

Children consolidate their knowledge of column multiplication.

They use these skills to solve multi step problems in a range of contexts.

Mathematical Talk

What is important to remember as we begin multiplying by the tens number?

How would you draw the calculation?

- Can the inverse operation be used?
- Is there a different strategy that you could use?

Varied Fluency



Calculate

4	4267		3046
×	34	×	73

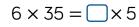
5734 × 26 =



Lauren made cookies for a bake sale. She made 345 cookies. The recipes stated that she should have 17 chocolate chips in each cookie. How many chocolate chips will there be altogether?

3

Work out the missing number.



Multiply 4-digits by 2-digits

Reasoning and Problem Solving

True or false.

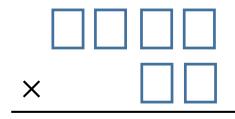
- a) 5,463 × 18 is the same as 18 × 5,463
- b) I can find the answer to 1,100 × 28 by using 1,100 × 30 and taking away two lots of 1,100

c) $70 \div 10 = 700 \div 100$

- a) True because multiplication is commutative so the calculation can be done in any order
- b) True because they both show 28 lots of 1,100
- c) True because both numbers have been made 10 times bigger



Place the digits in the boxes to make the largest product.



8432 × 75

Short Division

Notes and Guidance

Children build on their understanding of dividing up to 4 digits by 1 digit by now dividing by up to 2 digits. They use the short division and focus on division as grouping.

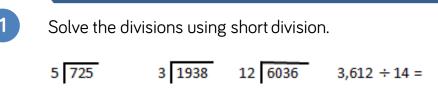
Teachers may encourage children to list the multiples of the number to help them solve the division more easily.

Mathematical Talk

What is different between dividing by 1 digit and 2 digits? If the number does not divide into the ones, what do we do?

Do we need to round our remainders up or down? Why does the context affect whether we round up or down?

Varied Fluency



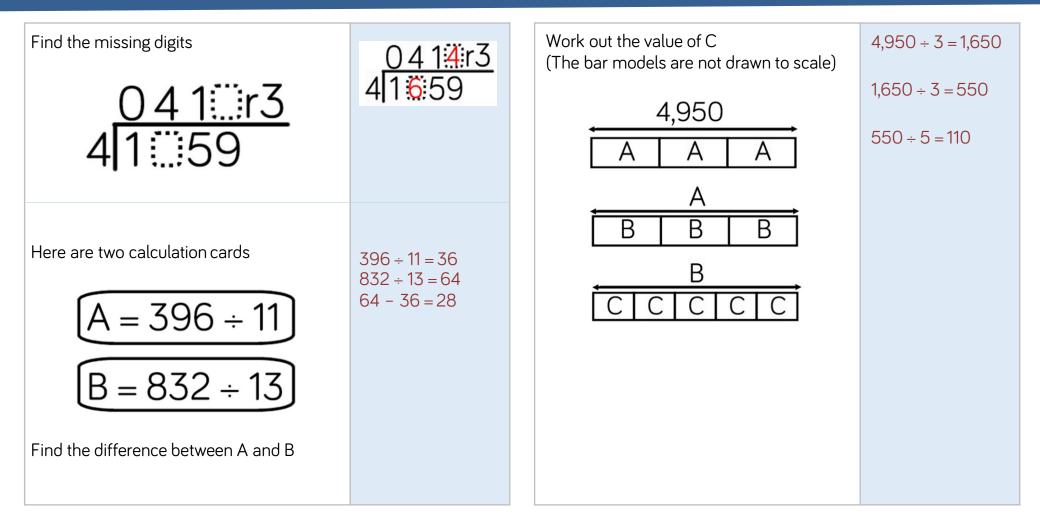
List the multiples of the number to helpyou calculate.

- 2
- A limousine company allows 14 people per limousine. How many limousines need to be hired for 230 people?
- 3

Year 6 have 2,356 pencil crayons for the year. They put them in bundles with 12 in eachbundle. How many complete bundles can be made?

Week 3 to 6 – Number: Four Operations

Short Division



Division using Factors

Notes and Guidance

Children need to use their number sense, specifically their knowledge of factors to be able to see relationships between the divisor and dividend. Beginning with multiples of 10 and moving on will allow the children to see the relationship before progressing forward.

Mathematical Talk

What is a factor?

How does using factor pairs help us to answer division questions?

- Do you notice any patterns?
- Does using factor pairs always work?
- Is there more than one way to solve a calculation using factor pairs?
- What methods can be used to check your working out?

Varied Fluency

780 ÷ 20 = 39 is the same as 780 ÷ 10 = 78 then 78 ÷ 2 = 39

What do you notice?

Use the same method to solve $480 \div 60$



Use factors to help you to answer

4,320 ÷ 15



Eggs are put into boxes holding a dozen. A farmer wants to put 648 eggs into boxes. How many boxes will he have filled?



Week 3 to 6 – Number: Four Operations

Division using Factors

Divide 1,248 by • 48 • 24 • 12 What did you do each time? Explain your strategy.	1,248 \div 48 = 26 1,248 \div 24 = 52 1,248 \div 12 = 104 I used factor pairs to complete the first question e.g. I divided 1,248 by 12 then divided the answer by 4 Because 24 is half of 48, I doubled 26 to get 52 I repeated this with 12 to get 104	Class 6 are solving 7,848 ÷ 24 The children decide which factor pairs to use between: • 2 and 12 • 4 and 6 • 10 and 14 Which will not give them the correct answer? Why?	10 and 14 will not give them the correct answer because 10 and 14 are not factors of 24	
Ivan To work out 4,320 ÷ 15 I will first divide 4,320 by 5 then divide the answer by 10 Is Ivan correct? Explain why.	Ivan is incorrect. He has partitioned 15 when he should have used factor pairs e.g. 5 and 3 The answer is 288			

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Week 3 to 6 – Number: Four Operations

Long Division (1)

Notes and Guidance

Children are introduced to long division as a different method of dividing by a 2-digit number. They divide 3digit numbers by a 2-digit number without remainders moving from a more expanded method with multiples shown to the more formal long division method.

Mathematical Talk

How can we use our multiples to help us divide by a 2-digit number?

Why are we subtracting the totals from the beginning number (seeing division as repeated subtraction)?

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided)

Varied Fluency

1			_		3	36		Multiples to help
	1	2		4				$12 \times 1 = 12$
				3	6	0	(×30)	$12 \times 2 = 24$
			_			2	(×6)	$12 \times 5 = 60$
2			_			2		$12 \times 10 = 120$

2

Solve the following divisions using Sam's method. Write out your multiples that may help you.

 $765 \div 17 = 450 \div 15 = 702 \div 18 =$

				0	3	6		
3	1	2		4	3	2		-
		2	 _	3	6		Ļ	_
					7	2		•
			-		7	2		
						0		-

Use the long division method to solve the following calculations. One has been done for you as an example.

836 ÷ 11=
798 ÷ 14 =
608 ÷ 19 =

Week 3 to 6 – Number: Four Operations

Long Division (1)

Reasoning and Problem Solving

Which calculation could be the odd one out below?

- 512 ÷ 16 =
- 672 ÷ 21=
- 928 ÷ 29 =
- 792 ÷ 24 =

Explain why.

512 ÷ 16 = 32	
672 ÷ 21 = 32	
928 ÷ 29 = 32	
792 ÷ 24 = 33	

Possible answers: 928 ÷ 29 is the odd one out because it is the only 3-digit number without a 2 in the ones column.

 $792 \div 24$ is the odd one out because it is does not have the answer 32 Explain the mistake

Instead of writing 10 lots of 16 as 160 they have written 10 lots of 16 as 106 This is therefore the mistake in the calculation.

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Week 3 to 6 – Number: Four Operations

Long Division (2)

Notes and Guidance

Building on using long division with 3 digit numbers, children divide four digit numbers by 2 digits using the long division method.

They use their knowledge of multiples and multiplying and dividing by 10 and 100 to calculate more efficiently.

Mathematical Talk

How can we use our multiples to help us divide by a 2-digit number?

Why are we subtracting the totals from the beginning number (seeing division as repeated subtraction)?

In long division, what does the arrow represent? (The movement of the next digit coming down to be divided)

Varied Fluency

			0	4	8	9			Here is a division
1	5		7	3	3	5			method. Solve the divisions using this
		_	6	0	0	0		(× 400)	method.
			_	-	3	-		(> 90)	2,208 ÷ 16 =
		_	1	2	0	0		(× 80)	
				1	L 3	5			1,755 ÷ 45 =
		-		1	L 3	5		(× 9)	= 1,536 ÷ 16
		_					0		



There are 2,028 footballers in tournament. Each team has 11 players and 2 substitutes. How many teams are in the tournament?



Week 3 to 6 – Number: Four Operations

Long Division (2)

Reasoning and Problem Solving

Which question is easier and which is harder?

- 1,950 ÷ 13 =
- 1,950 ÷ 15 =

Explain why.

1,950 ÷ 13 is harder because 13 is a prime number and therefore cannot be split into factors and divided in smaller parts.

6,823 ÷ 19 = 359 r2 8,259 ÷ = 359 r2 Find the value of

= 23

Long Division (3)

Notes and Guidance

Children now divide using long division where their answers have remainders. After dividing, they check that their remainder is smaller than their divisor.

Children start to understand when rounding is appropriate to use for interpreting the remainder and when the context means that this is not applicable.

Mathematical Talk

How can we use our multiples to help us divide?

What happens if we cannot divide our ones exactly by our divisor? How do we show what we have left over?

Why are we subtracting the totals from the starting amount (seeing division as repeated subtraction)?

Does the remainder need to be rounded up or down?

Varied Fluency



Elijah uses this method to calculate 372 divided by 15. He has used his knowledge of multiples to help.

		2	4 r	1 2		1 × 15 = 15
1 5	3	7 2	2			2 × 15 = 30
	 - 3	0 0)		(× 20)	3 × 15 = 45
		-			(**==7	$4 \times 15 = 60$
		7 3	2			5 × 15 = 75
		6 0)		(× 4)	10 × 15 = 150

1 2

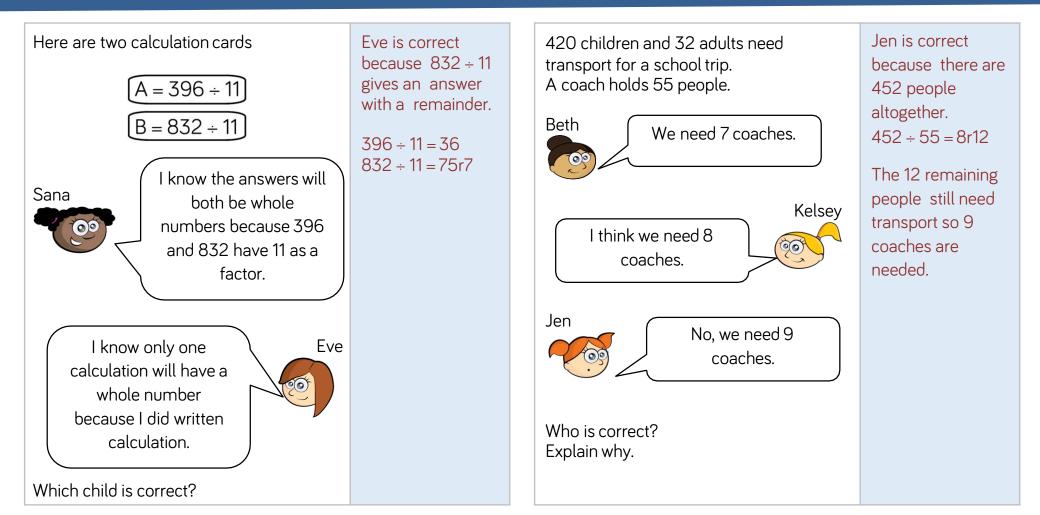
Solve the following calculations using Elijah's method. Show the multiples that you need to use to help you.

271 ÷ 17 = 623 ÷ 21 = 842 ÷ 32 =

2 A school needs to buy 380 biscuits to pass around at parents' evening. They come in packets of 12. How many packets with the school need to buy?

Week 3 to 6 - Number: Four Operations

Long Division (3)



Long Division (4)

Notes and Guidance

Children now divide four-digit numbers using long division where their answers have remainders. After dividing, they check that their remainder is smaller than their divisor.

Children start to understand when rounding is appropriate to use for interpreting the remainder and when the context means that this is not applicable.

Mathematical Talk

How can we use our multiples to help us divide?

What happens if we cannot divide our ones exactly by our divisor?

How do we show what we have left over?

Why are we subtracting the totals from the starting amount (seeing division as repeated subtraction)?

Does the remainder need to be rounded up or down?

Varied Fluency

Simon used this method to calculate 1426 divided by13. He wrote down his multiples key facts to help him work out the answer.

				1	0	9	r	9
1	3		1	4	2	6		-
		_	1	3	0	0		(×100)
		_	0	1	2	6		-
		_		1	1	7		(× 9)
		_		0) (9		-

Using Simon's method answer the following: $2,637 \div 16 =$ $4,231 \div 22 =$ $4,203 \div 18 =$



There are 7,849 people going to a concert. Each coach holds 64 people. How many coaches are needed to transport all the people?

Long Division (4)

Class 6 are completing this calculation $3,636 \div 12$	Violet is incorrect because the answer is 303	Using the number 4,236, how many numbers up to 20 does it divide by without a remainder?	1, 2, 3, 4, 6, 12 They are all factors
Violet Violet I know there will be a remainder before I start.	Violet could have partitioned the number into 3,600 and 36 to see that it is divisible by 12	Is there pattern?	of 12
Is she correct?			
Explain how you know.			

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Week 3 to 6 – Number: Four Operations

Common Factors

Notes and Guidance

Children find the common factors of two numbers. Some children may still need to use arrays and other representations at this stage but mental methods and knowledge of multiples should be encouraged. They can show their results using Venn diagrams and tables.

Mathematical Talk

- How do you know you have found all the factors of a given number?
- Have you used a system?
- Can you explain your system to a partner?
- How does a Venn diagram help to find common factors?
- Where are the common factors?

Varied Fluency

1

What are the common factors of these pairs of numbers?

- 24 and 36
- 20 and 30
- 28 and 45



Which number is the odd one out?

12, 30, 54, 42, 32, 48

Can you explain why?



Two numbers have common factors of 4 and 9

What could the numbers be?

Week 3 to 6 – Number: Four Operations

Common Factors

There are 49 apples and 56 pears. They need to be put into baskets with an equal number in each basket. Jamil Jamil I think there will be baskets with 8 pieces of fruit in each	There will be 7 pieces of fruit in each basket because 7 is a common factor of 49 and 56	Tom has 2 pieces of string. One is 160cm long and the other is 200cm long. He cuts them into pieces of equal length. What are the possible lengths the string could be?	2, 5, 10 and 20 are common factors of 160 and 200
I think there will be baskets with 7 pieces of fruit in each Who is correct? Explain how you know.		Tahil has 32 football cards that he is giving away to his friends. He shares them equally. How many friends could Tahil have?	1, 2, 4, 8 or 16 friends.

Common Multiples

Notes and Guidance

Building on knowledge of multiples, children find common multiples of numbers. They should continue to use a visual representation to support their thinking. They also use more abstract methods to calculate the multiples and use numbers outside of times tablefacts.

Mathematical Talk

Are the lowest common multiples of a pair of numbers always the product of them? Can you think of any strategies to work out the lowest common multiples of different numbers? When do numbers have common multiples that are lower than their product?

Varied Fluency

On a 100 square, shade the first 5 multiples of 7 and then the first 8 multiples of 5 What do you notice? Choose 2 other times tables which you think will have more than 3 common multiples.



List 5 common multiples of 4 and 3

3

Jim and Nancy play football at the same local football pitches. Jim has plays once every 4 days and Nancyplays once every 6 days. In a fortnight, how many times will they play football on the same day?

Common Multiples

diagram.	Headings: Multiples of 4 Multiples of 6 144 is a multiple of 6 and 8	Nancy is double her sister's age. They are both older than 20 and younger than 50 Their ages are both multiples of 7 Work out their ages.	Nancy: 42 Nancy's sister: 21
Can you think of a multiple of 6 and 8 that is a square number?		 Train starts running from Leeds to York at 7am. The last trains leaves at midnight. Platform 1 has a train leaving from it every 12 minutes. Platform 2 has one leaving from it every 5 minutes. How many times in the day would there be a train leaving from both platforms at the same time? 	Platform 1 and 2 will have a train leaving at the same time once every hour at o'clock. Therefore there will be 18 times from 7am to midnight when a train will leave at both platform 1 and 2

Primes to 100

Notes and Guidance

Building on their learning in year 5, children should know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers.

They should be able to use their understanding of prime numbers to work out whether or not numbers up to 100 are prime. Using primes, they break a number down into its prime factors.

Mathematical Talk

What is a prime number? What is a composite number? How many factors does a prime number have? Are all prime numbers odd? Why is 1 not a prime number? Why is 2 a prime number?

Varied Fluency



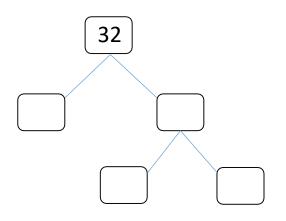
List all the prime numbers between 10 and 30



3

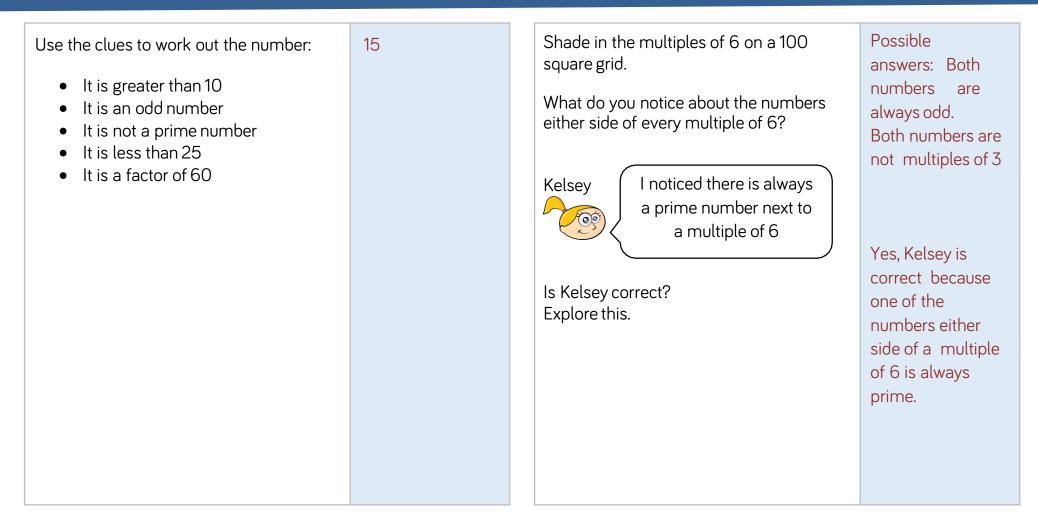
The sum of two prime numbers is 36. Which numbers are they?

All numbers can be broken down into prime factors. A prime factor tree can help us find them. Complete the prime factor tree for 32



Week 3 to 6 – Number: Four Operations

Primes to 100



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Square & Cube Numbers

Notes and Guidance

- Children have identified squared and cubed numbers previously and now need to explore the relationship between them and solve problems involving these numbers.
- They need to experience sorting the numbers into different diagrams and look for patterns and relationships. they need to explore general statements.
- This step is a good opportunity to practise efficient mental methods of calculation.

Mathematical Talk

- What do you notice about the sequence of square numbers?
- What do you notice about the sequence of cube numbers?
- Explore the pattern of difference between the numbers.

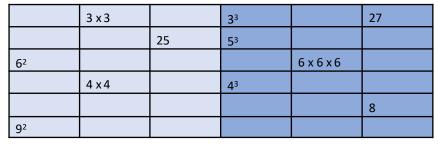
Varied Fluency

1

Use symbols \leq , \geq or = to make these statements correct

3 cubed	6 squared
8 squared	4 cubed
11 squared	5 cubed

- 2
- This table shows squared and cubed numbers. Complete the table. Explain the relationships you can see between the numbers.



+

210 -

3

+ 35 = 99

= 41

Which square numbers are missing from the calculations above?

Square & Cube Numbers

Reasoning and Problem Solving

Place 5 odd and 5 even numbers in the diagram below.

	Not cubed	Cubed
Over 100		
100 or less		

Put at least one number in each section.

Possible cube numbers to use: 8, 27, 64, 125, 216, 343, 512, 729, 1,000 Shade in all the square numbers on a 100 square grid.

Now shade in multiples of 4 on a 100 square grid.

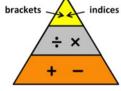
What do you notice?

Square numbers are always either a multiple of 4 or one more than a multiple of 4

Order of Operations

Notes and Guidance

Children will look at different operations within a calculation and consider how the order of operations affects the answer. The following image is useful when referring to the order of operations.



Mathematical Talk

Does it make a difference if you change the order in a mixed operations calculation?

What would happen if we did not use the brackets?

Would the answer be correct?

Varied Fluency

Sarah had 7 bags with 5 sweets in each. She added one more to each bag. Circle the calculation below that shows the correct working out.

7 (5 + 1) = 42 $7 \times 5 + 1 = 36$ $7 \times 5 + 1 = 42$

2

Daniel completed the following calculation and got the answer 168

 $2(30 \div 5) + 14 = 168$

Can you explain what he did and where he made the mistake?

3

Add brackets and the missing numbers to complete

$$3 + \bigcirc \times 5 =$$

25 - 6 × $\bigcirc =$

Why?

Order of Operations

 Play Countdown. Big numbers: 25, 50, 75, 100 Small numbers: 1 - 10 Without looking at the number cards, children choose 6 cards from across the big and small number cards. Reveal a target number. Children aim to make the target number. 	Possible example: Cards chosen: 75, 25, 2, 5, 6, 10 Target number: 458 Calculation: $10 - 2 + (75 \times 6)$	 Write different number sentences using the digits 3, 4, 5 and 8 before the equals sign that use: One operation Two operations, no brackets Two operations with brackets 	Possible answers: 58 - 34 = $58 + 3 \times 4 =$ 5(8 - 3) + 4 =
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Mental Calculations

Notes and Guidance

We have included this small step separately to ensure that teachers give emphasis to this important skill. Discussions around efficient mental calculations and sensible estimations need to run through all steps.

Sometimes children are too quick to move to computational methods, when changing the order leads to quick mental methods and solutions.

Mathematical Talk

Is there an easy and quick way to do this?

Can you use known facts to answer the problem?

Can you use rounding?

Does the solution need an exact answer?

How does knowing the approximate answer help with the calculation?

Varied Fluency

How could you change the order of these calculations to be able to perform them mentally?

50 x 16 x 2= 30 x 12 x 2= 25 x 17 x 4=

2

Jamie buys a t shirt for £9.99, socks for £1.49 and a belt for £8.99 He was charged £23.47

How could he quickly check if he was overcharged?

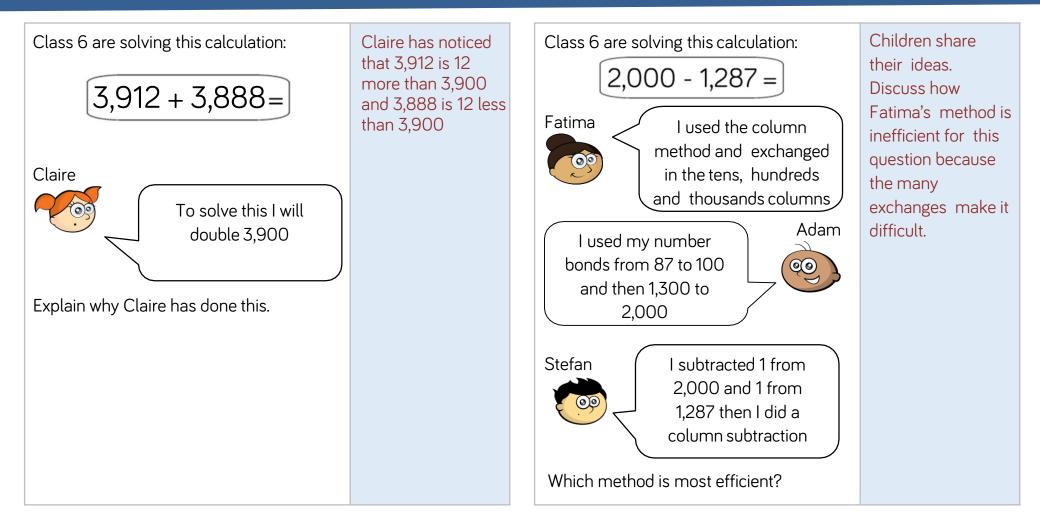




What do you estimate that B represents when: A = 0 and C = 1,000 A = 30 and C = 150 A = -7 and C = 17 A = 0 and C = 5,000A = 1,000 and C = 100,000

Week 3 to 6 – Number: Four Operations

Mental Calculations



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Reason from Known Facts

Notes and Guidance

Pupils should be able to use their understanding of known facts from one calculation to work out the answer of another similar calculation without starting afresh.

They should use reasoning and apply their knowledge of commutativity and inverse.

Mathematical Talk

- What is the inverse?
- When do you use the inverse?
- How can we use multiplication/division facts to help us answer similar questions?

Varied Fluency

$$70 \div = 3.5$$

$$70 \div = 7$$

$$- \div 2 = 35$$

$$- \times 3.5 = 7$$

$$3.5 \times 20 = -$$

$$70 \div = 3.5$$

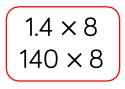
Make a similar set of calculations using $90 \div 2 = 45$



5138 ÷ 14 = 367 Use this to work out 15 × 367



 $14 \times 8 = 112$ Use this to work out:



Reason from Known Facts

