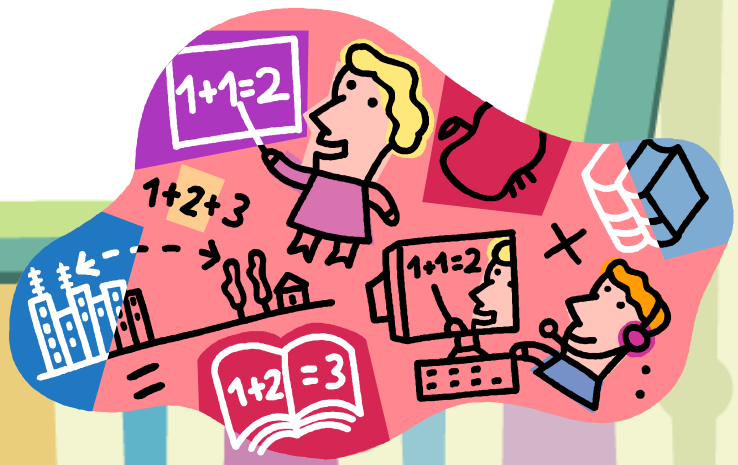
Maths

- How do we teach it?
- What do the written methods look like?
- What problem solving strategies do we use?
- What can you do at home to help?



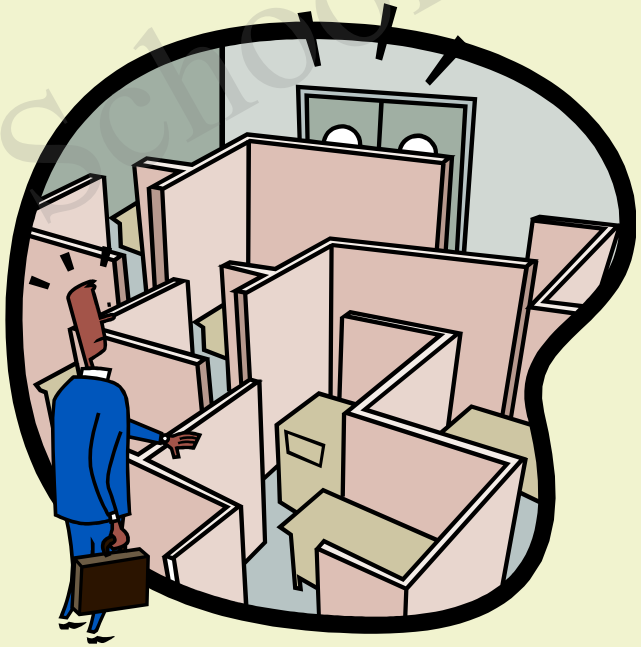
Game

Ping Pong!

You begin by setting a rhythm where one person, i·e· the adult says ping and the children reply pong· You then start to give a number in place of ping and the children have to give the complement to 10 in place of pong, so it would go something like this 'ping, pong, ping, pong, six, four, ping, pong, five, five' etc·

Children really enjoy it and can actually hear themselves getting quicker at responding·

‘It just doesn’t look like it did in my day.’



‘I hear and I forget. I see and I remember.
I do and I understand.’
(A Chinese proverb)

Do we all need to be able to work out **27 x 43** precisely with a pen and paper?

Probably not...but we do need to know that:

27 x 43 is roughly 30 x 40 and...

that this is roughly 1,200

It's partly the need to have a good feel for numbers that is behind the modern methods.

The Aim

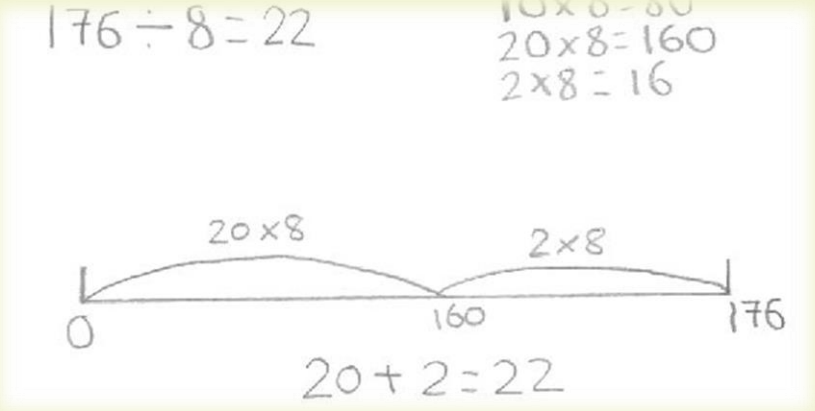
- For children to do mathematics in their heads, and if the numbers are too large, to use pencil and paper to avoid losing track.
- To do this children need to learn quick and efficient methods, including mental methods and <u>appropriate</u> written methods.

Learning written methods is not the ultimate aim.

Mathematics is foremost an activity of the mind; written calculations are an aid to that mental activity.

We want children to ask themselves:

1. Can I do this in my head?
2. Can I do this in my head using drawings or jottings?
3. Do I need to use an expanded or compact written method?
4. Do I need a calculator?



Singapore Maths

Children can under perform in mathematics because they find it boring or they can't remember all the rules.

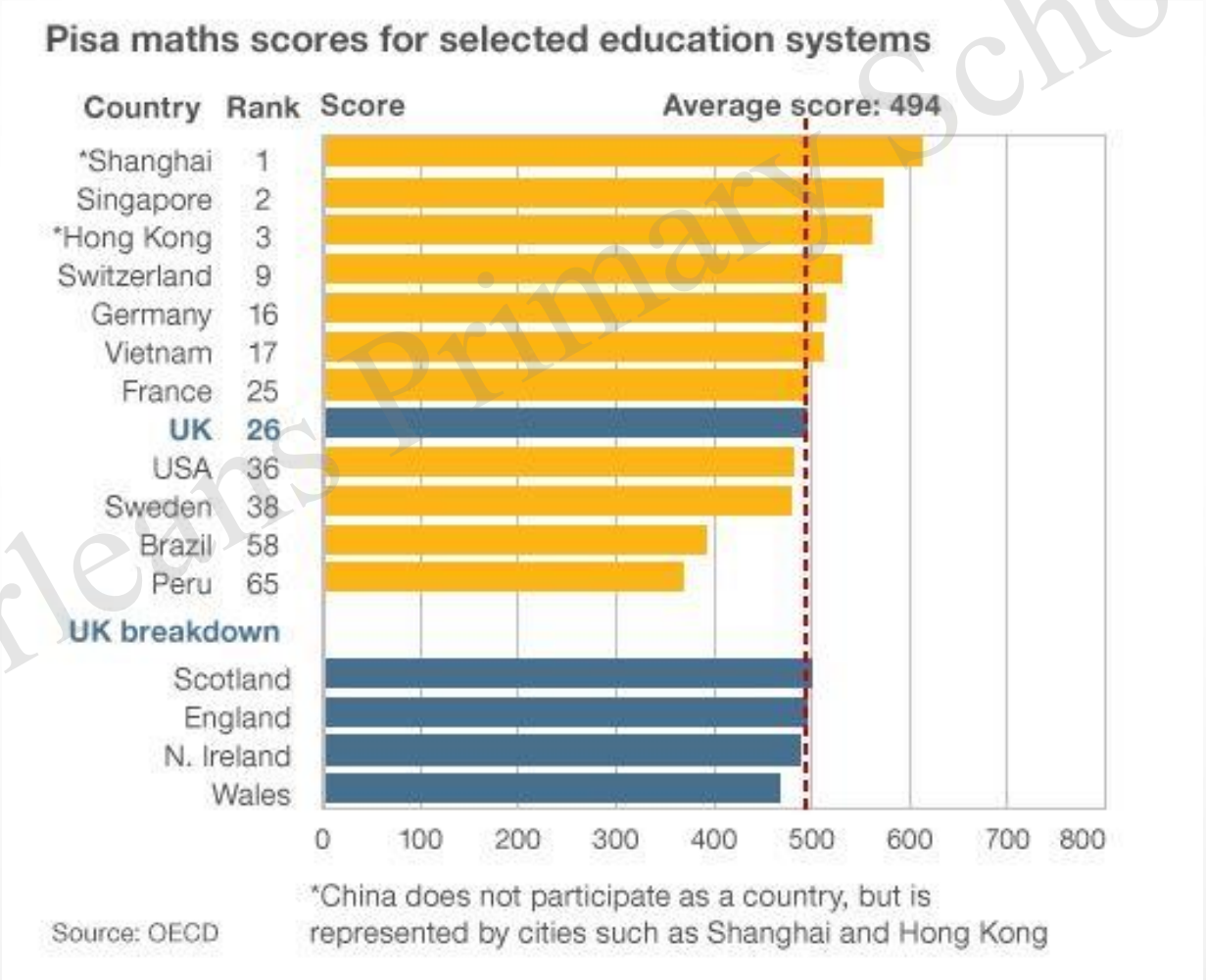
The Singapore method of teaching mathematics develops pupils' mathematical ability and confidence without having to resort to memorising procedures to pass tests - making mathematics more engaging and interesting.

All in agreement...

Ofsted, the National Centre for Teaching Mathematics (NCETM), the Department for Education, and the National Curriculum Review Committee have all emphasised the pedagogy and heuristics used by Singapore. This method is now being used successfully in the UK by the Ark academies, the Harris Federation, Primary Advantage as well as numerous state, free, and independent schools.

Results...

Singapore Maths has produced a world-class level of achievement for years. Singapore students scored first in the past three "Trends in International Mathematics and Science Studies" (TIMSS).



Features of Singapore Maths:

- Emphasis on problem solving and comprehension, allowing children to relate what they learn and to connect knowledge
- Careful scaffolding of core competencies of :
  - visualisation, as a platform for comprehension
  - mental strategies, to develop decision making abilities
  - pattern recognition, to support the ability to make connections and generalise
- Emphasis on the foundations for learning and not on the content itself so students learn to think mathematically as opposed to merely reciting formulas or procedures.

The Perfect Model: CPA

One of the key learning principles behind Singapore maths is the concrete pictorial abstract approach, often referred to as CPA.

The concrete-pictorial-abstract approach, based on research by psychologist Jerome Bruner, suggests that there are three steps necessary for pupils to develop understanding of a concept.

Concrete representation

A child is first introduced to an idea or a skill by acting it out with real objects. In division, for example, this might be done by separating apples into groups of red ones and green ones or by sharing 12 biscuits amongst 6 children. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

Pictorial representation

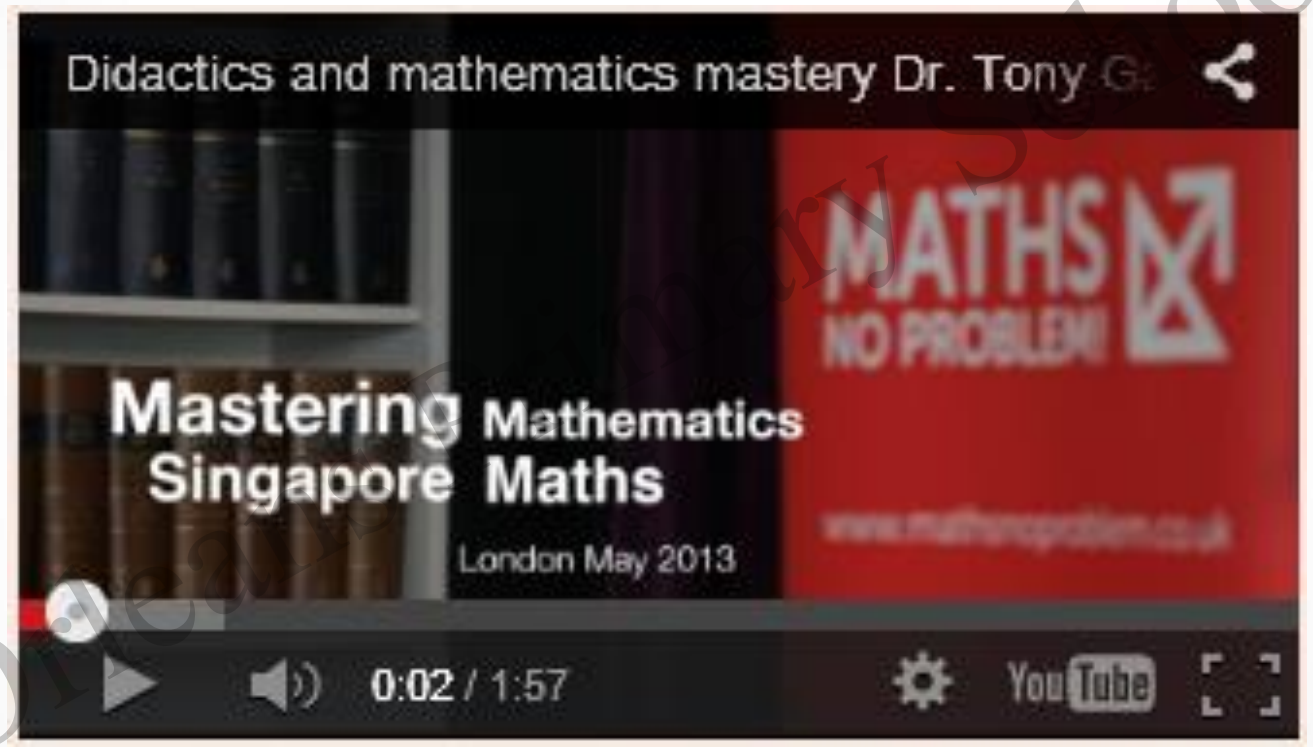
A child has sufficiently understood the hands-on experiences performed and can now relate them to representations, such as a diagram or picture of the problem. In the case of a division exercise this could be the action of circling objects.

Abstract A child is now capable of representing problems by using mathematical notation, for example: 12 ÷ 2 = 6 This is the ultimate mode, for it "is clearly the most mysterious of the three."

- CPA Video

concrete	Representational	Abstract
① 		4+5=9
② 		

Think about this...



First steps in Maths

- The ability to calculate mentally forms the basis of all calculations and therefore, up until the end of Year 3, the focus in class is on teaching these mental methods.

- Throughout KS1 and the beginning of KS2, pupils build up their counting strategies and develop a secure understanding of place value and number facts. They use a range of practical activities and are encouraged to jot things down informally or use resources to help them.

- As children become more confident, their mental strategies will increase in speed and accuracy and they may no longer need resources to help them

What's next?

- It is only once the children are confident in these mental strategies that they begin to look at more formal written methods.
- As they move through KS2, the children are taught a number of techniques, initially expanding on their mental strategies before progressing towards an efficient and more compact written method.

Our Aim

- Our ultimate aim is to ensure that all children have a reliable written method with which they feel confident using when they are unable to carry out the calculation mentally.

- It is important to understand that our brains don't all function in the same way - different children (and adults) will prefer to use different methods. By teaching a wide range of calculation strategies, children are given the opportunity to choose one that they understand and feel comfortable with, which ultimately increases their confidence and accuracy.

Mental Methods

Oral and mental skills are practised, rehearsed and built on in all year groups.

Secure mental calculation requires the ability to:

☐ recall key number facts instantly, e.g. number bonds to 10, 100 etc and multiplication facts up to 12 x12

☐ use knowledge of place value to extend known number facts – e.g. using the fact that 7 x 6 = 42 to calculate 70 x 6 = 420;

☐ use strategies such as realising that addition or multiplication can be done in any order, or partitioning numbers into hundreds, tens and ones.

By becoming confident in a selection of different methods, pupils are encouraged to tackle problems by finding the most reliable, efficient way of approaching a calculation. For example, if the question requires you to find 800 – 198, it may be best to count on from 198 to 200, and then on to 800 rather than count backwards from 800.

Step 1

Oral and mental skills are practised, rehearsed and built on in all year groups.

In the very early years, the focus is on developing an understanding of numbers. We start by teaching the children to count on and back, at first with their fingers, then with number lines and hundred squares. They will do lots of practical activities which involve counting objects and will learn about the terms 'more' and 'less'.

Step 2

Once children have gained an understanding of what numbers actually are, the focus becomes number bonds to 10 and then 20 (e.g. 1 + 9; 2 + 8; 6 + 4 etc.)

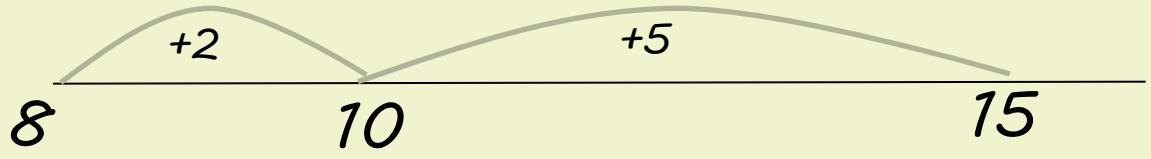
Again, this is taught using a variety of practical methods until the children are confident enough to do them mentally.

Step 3

Children need to be able to recall other simple addition facts such as 4 + 5 and 8 + 9 and relate this to multiples of 10 and 100 (e.g. 40 + 50; 80 + 90).
These are secured through sufficient practice.

When learning to add numbers such as 8 + 7 or 26 + 7, they will learn to 'bridge through 10' to make adding quicker and more efficient. So they will make the first number add to the next 10, then add on what's left. For example,

For 8 + 7, they will do 8 + 2 = 10,
then add on the rest: + 5 = 15.



Step 4

It is also crucial that children gain an understanding of place value (how much each digit in a number is worth) and how to partition a number into ones (units), tens, hundreds etc. This helps the children manage the calculation more easily.

For example:

23 + 46

can be broken down into:

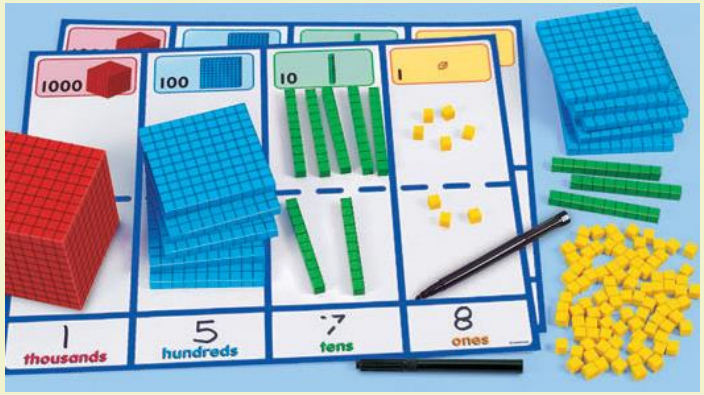
20 + 40 = 60

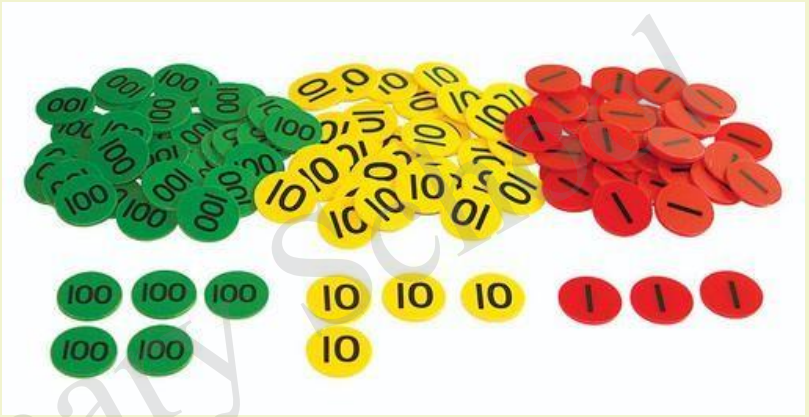
3 + 6 = 9

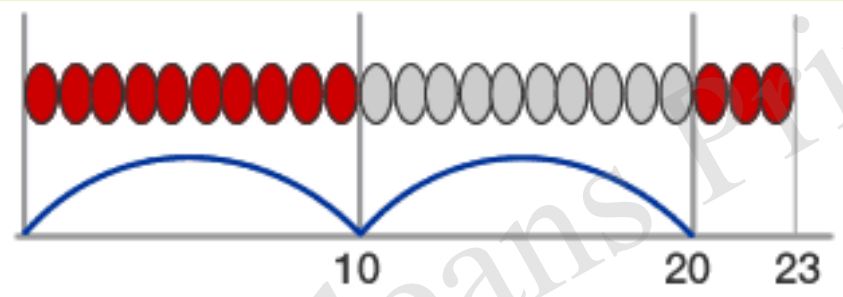
60 + 9 = 69

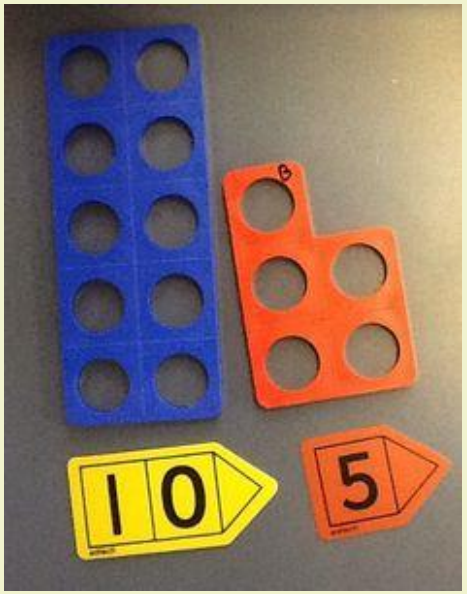
They can then use their knowledge of place value to extend known number facts - e.g. using the fact that 7 x 6 = 42 to calculate 70 x 6 = 420.

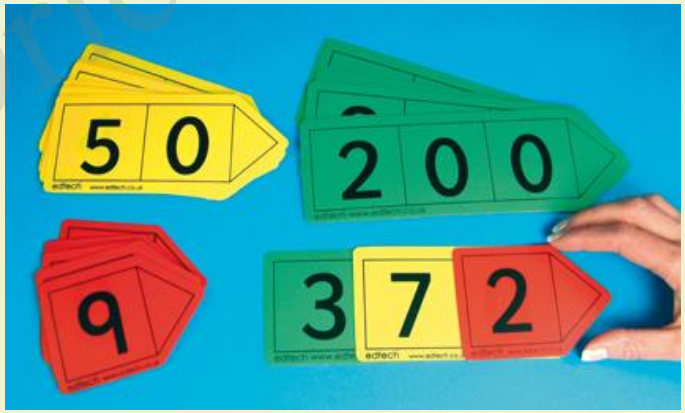
Place Value





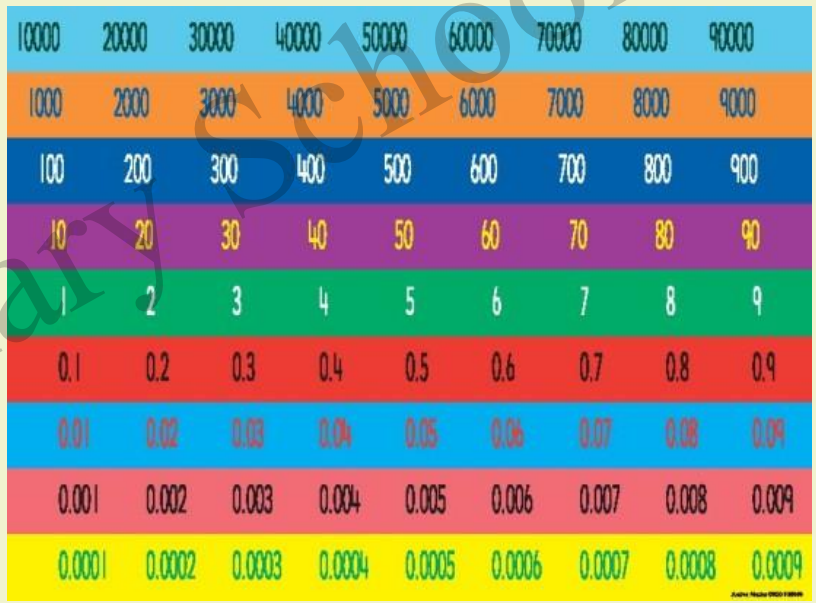






Place Value

TH	H	T	U



hundreds	tens	ones		tenths	hundredths	thousandths
4	5	2	.	7	8	1

Let's do some Maths!

ADDITION

Key Words

total come increase

both altogether

sum plus combine

in all add join

How would you solve these?

- 25 + 42
- 25 + 27
- 25 + 49
- 78 + 199
- 145 +127

A sledgehammer to crack a nut!

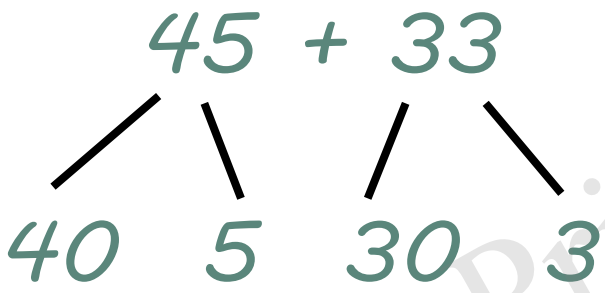
$$\begin{array}{r} {}^{0}\cancel{1}\,{}^{1}\cancel{0}\,{}^{9}\cancel{0}\,{}^{9}\cancel{0}\,{}^{1}0 \\ -\quad\quad 7 \\ \hline 9\,9\,3 \end{array}$$

$$\begin{array}{r} {}^{1}\cancel{1}6\,{}^{0} \\ -\ 9 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 97 \\ \times\ 100 \\ \hline 00 \\ 000 \\ 9700 \\ \hline 9700 \end{array}$$

$$7\overline{)\,{}^{0}\cancel{5}\,6}\quad 08$$

Stage 1: Partitioning



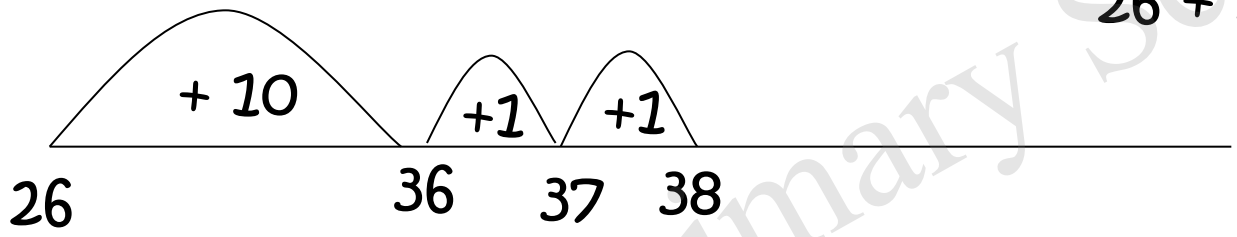
40 + 30 = 70
5 + 3 = 8
70 + 8 = 78

Your turn...

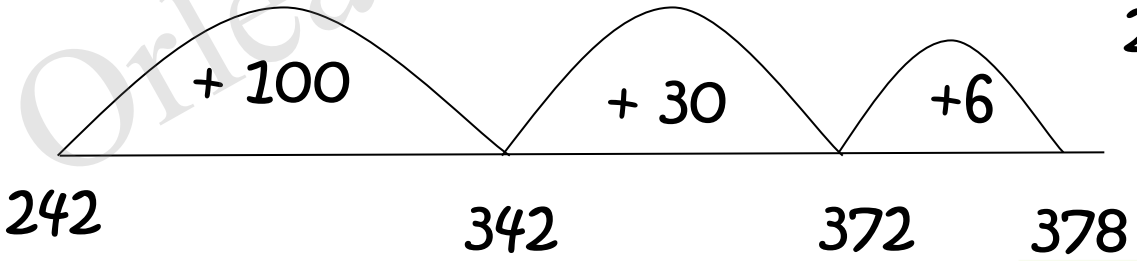
53 + 46 =

Stage 2: Number line

26 + 12 = 38

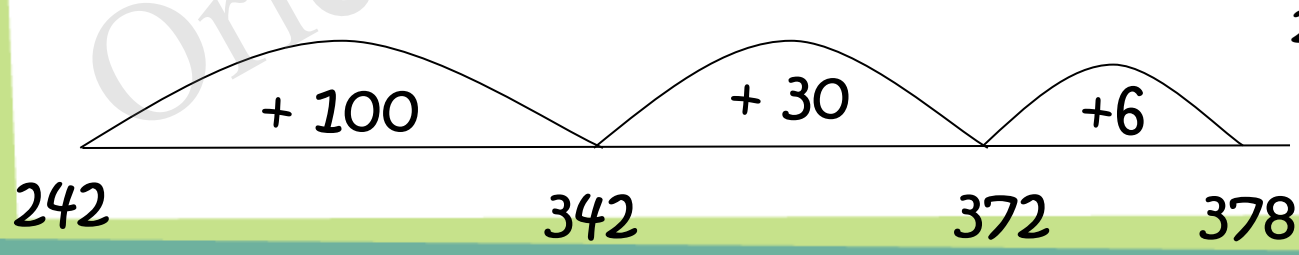


242 + 136 = 378



Use the number line to work these out...

- 67 + 48 =
- 346 + 237 =
- 3241 + 1471 =



242 + 136 = 378

Stage 3: Column Method

Expanded

$$
\begin{array}{r}
358 \\
+\ 33 \\
\hline
11 \\
80 \\
300 \\
\hline
391
\end{array}
$$

Leading to →

Compact

$$
\begin{array}{r}
358 \\
+\ 33 \\
\hline
391 \\
{\scriptstyle 1}
\end{array}
$$

SUBTRACTION

Key Words

Take away

decrease

remain

difference

minus

How many more?

fewer

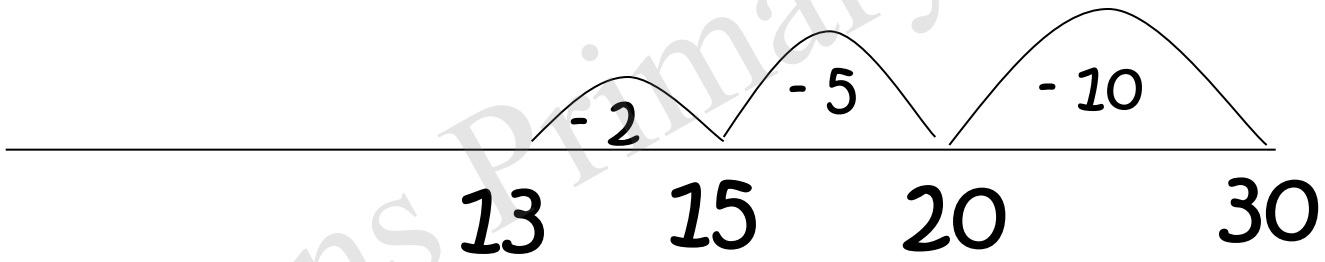
left

How would you solve these?

- 67 - 45
- 67 - 59
- 178 - 99
- 3241 - 2167

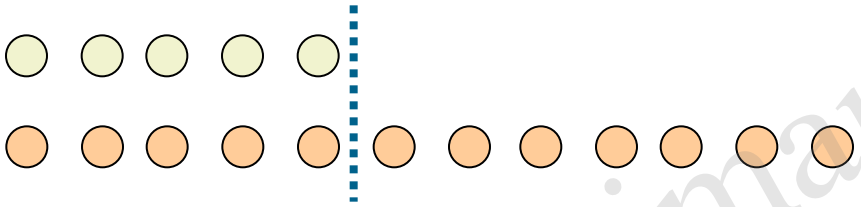
Stage 1: Number line

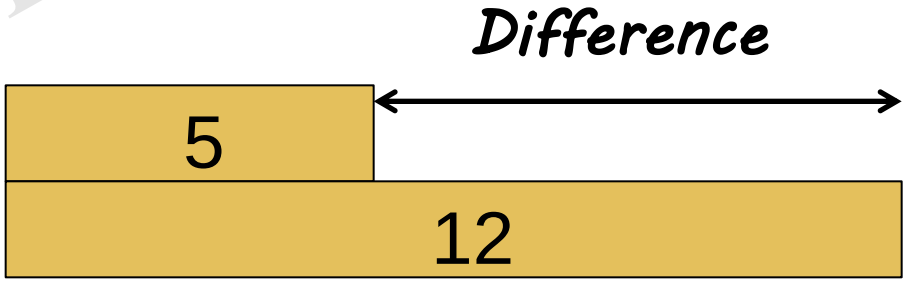
- Subtraction as taking away



30 – 17 = 13

- Subtraction as finding the difference

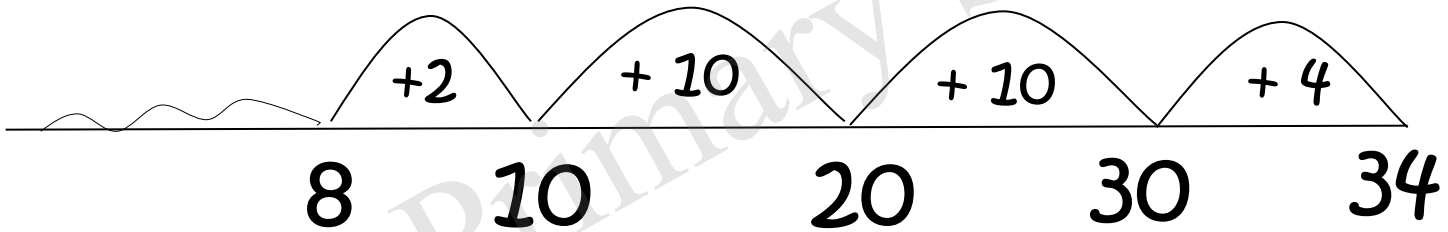




Stage 2: Number line

- Subtraction as finding the difference

34 – 8 =

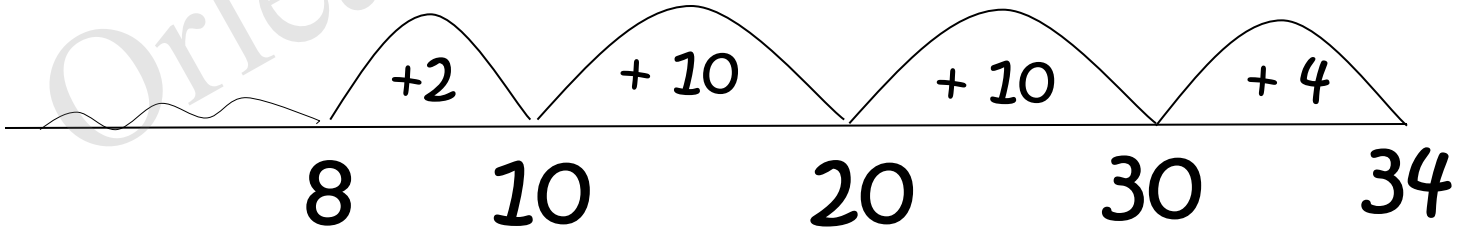


- Jump to next multiple of 10
- Count the jumps

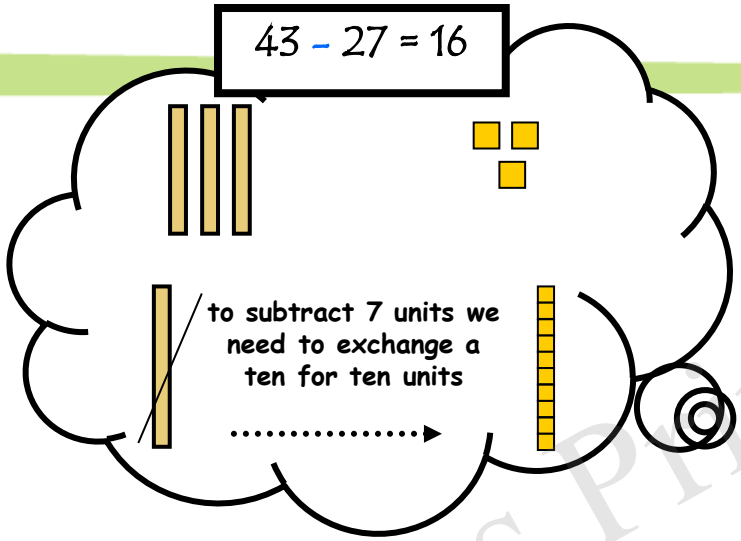
10 + 10 + 4 + 2 = 26

Use the number line to work these out...

- 48 - 31 =
- 256 - 107 =



Stage 3: Expanded Column Method



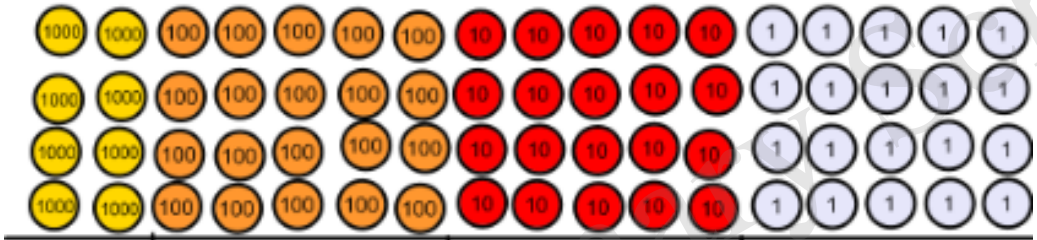
Expanded method

It is important that the children have a good understanding of place value and partitioning using concrete resources and visual images to support calculations. The expanded method enables children to see what happens to numbers in the standard written method.

T	U
4	3
– 2	7

$$\begin{array}{rrcr} {}^{30}\cancel{40} & & + & {}^{10+}3 \\ - & 20 & + & 7 \\ \hline & 10 & + & 6 \end{array}$$

Exchange Game



Thousands	Hundreds	Tens	Ones
Total:			

Stage 4: Column Method

$$\begin{array}{r} 547 \\ -\ 134 \\ \hline 413 \end{array}$$

$$\begin{array}{r} {}^{3}\cancel{4}{}^{1}3 \\ 27 \\ \hline 16 \end{array}$$

Multiplication

per

of

multiplied

product

twice

double

by

as much

times

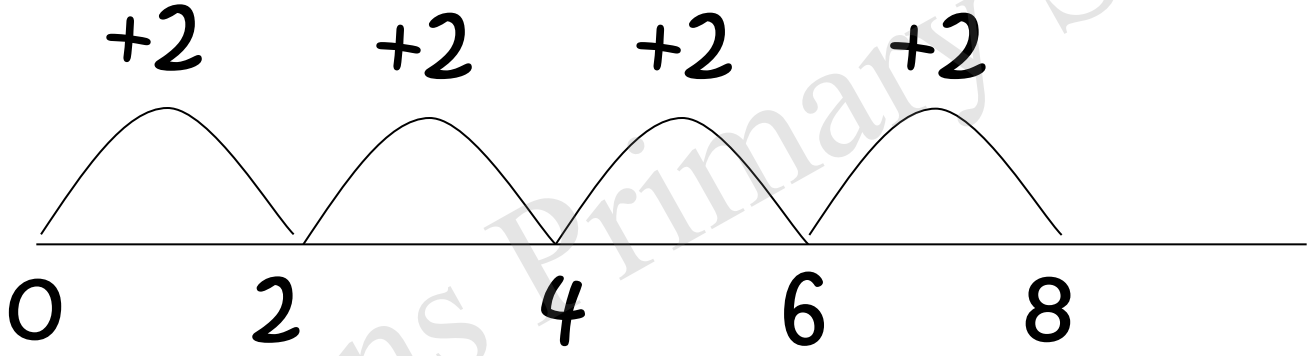
How would you solve these?

- 24 × 50
- 24 × 4
- 24 × 15
- 136 × 9

Stage 1: Number line

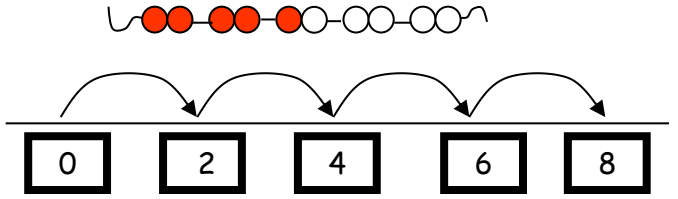
Multiplication as repeated addition

4×2



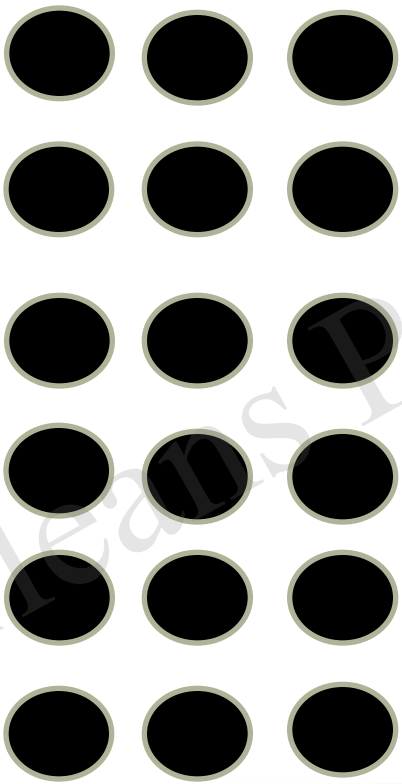
$4 \times 2 = 2 + 2 + 2 + 2$

So, $2 \times 4 = 8$

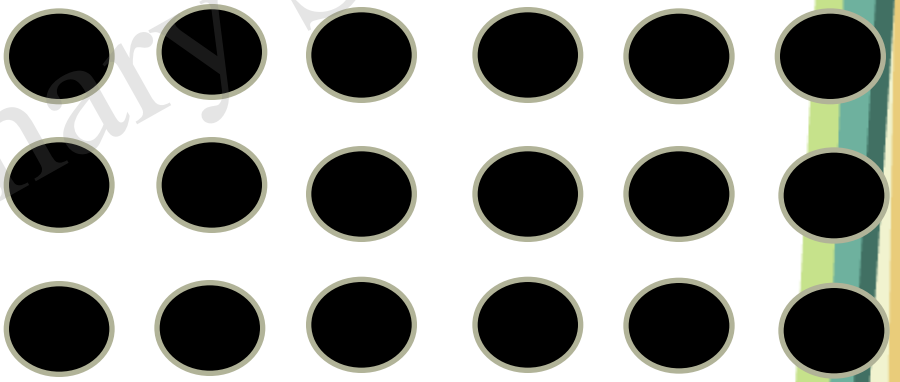


Stage 2: Arrays

3 x 6



Or



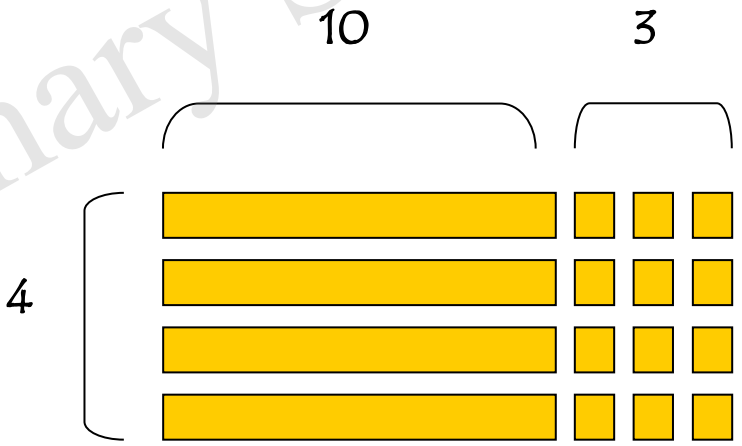
Add the dots

What multiplication are these arrays showing?

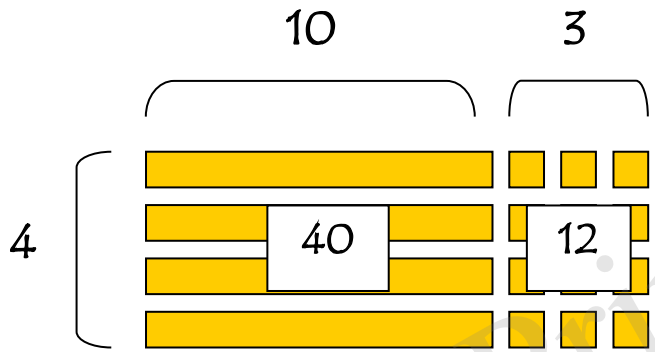
Stage 3: Partitioning

Use place value apparatus to support the multiplication of U x TU

4 x 13



Stage 3: Partitioning



	10	3
4	40	12

40 + 12 = 52

Use place value apparatus to support the multiplication of U x TU alongside the grid method

4 x 13

Stage 3: Partitioning

24 x 5

20 x 5 = 100
4 x 5 = 20

100 + 20 = 120

Step 4: Grid Method

Multiplying TU x TU

14 x 33

	30	3
10	300	30
4	120	12

= 330 +

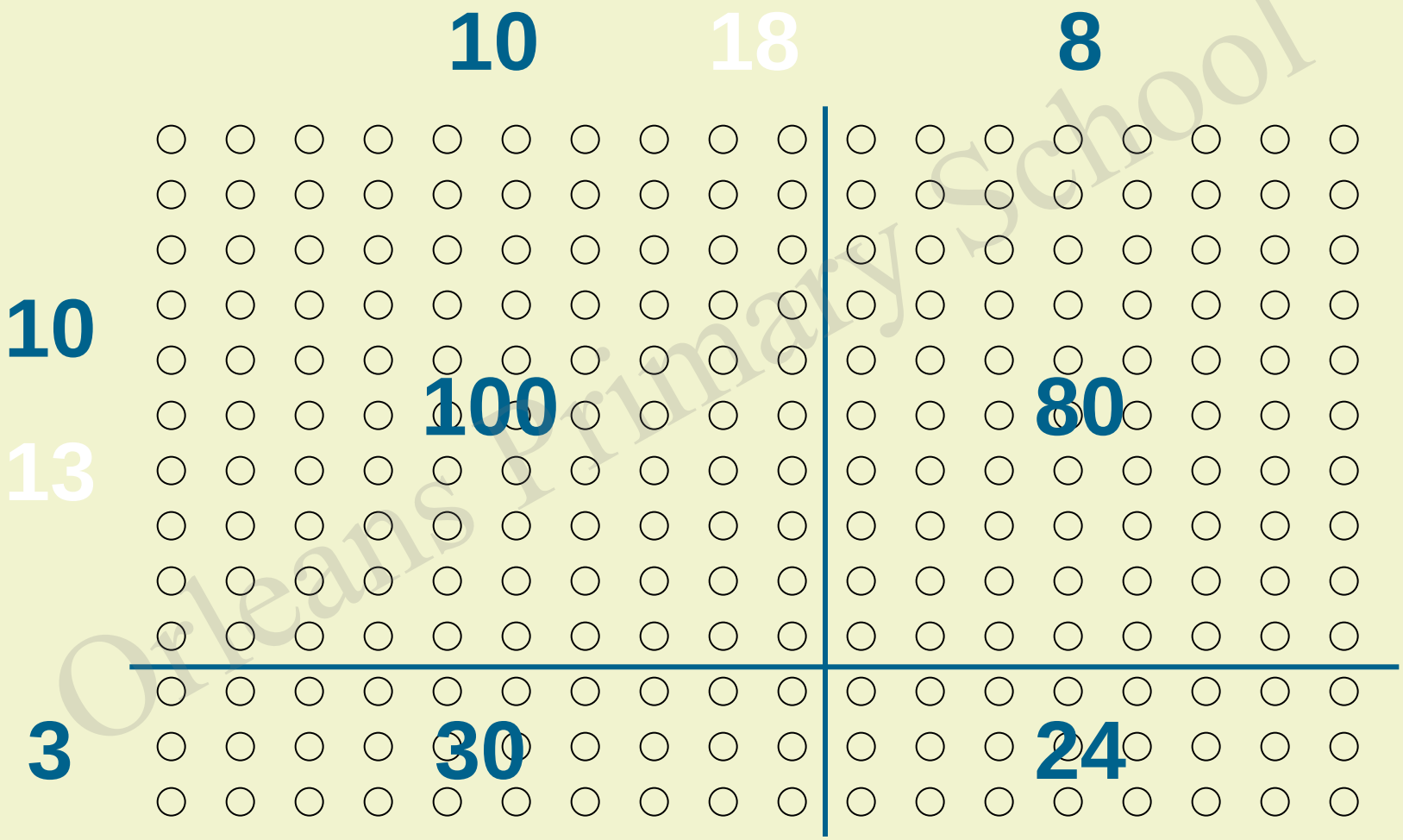
= 132

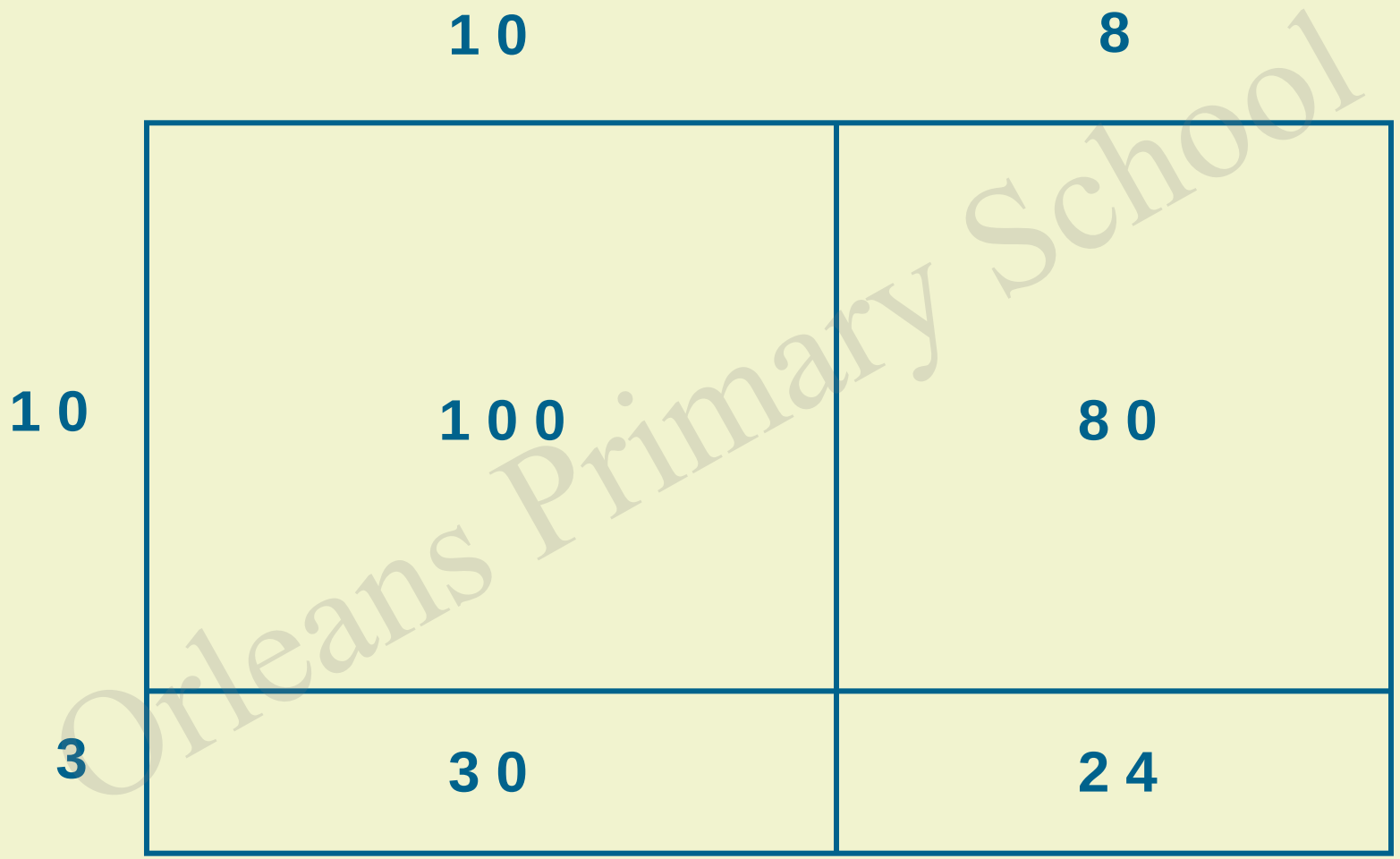
462

BBC News Video Link

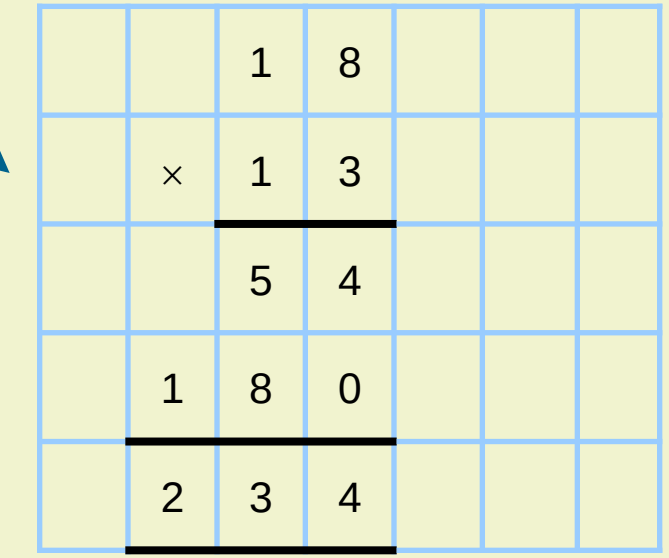
Stage 5: Expanded Method

$$
\begin{array}{rl}
 & 38 \\
\times & 7 \\
\hline
 & 210 \quad (30 \times 7) \\
 & 56 \quad (8 \times 7) \\
\hline
 & 266
\end{array}
$$





	10	8
10	100	80
3	30	24



Use the grid method to work these out:

24 x 7

142 x 3

14 x 33

	30	3
10	300	30
4	120	12

= 330 +

= 132

462

Division

cut	each
evenly	Every
Quotient	out of
equal parts	divided by
In half	average

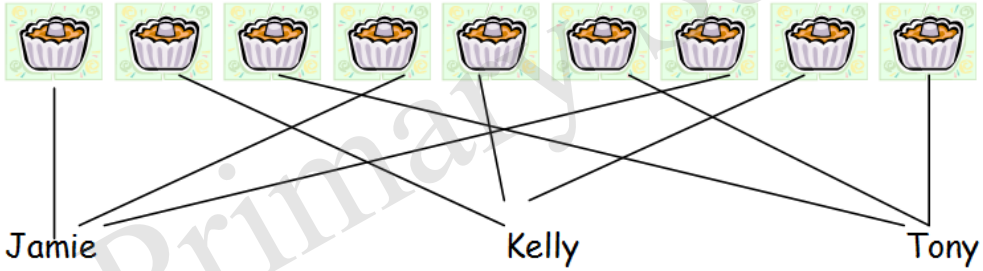
How would you solve these?

- 123 ÷ 3
- 165 ÷ 10
- 325 ÷ 25
- 623 ÷ 24

Division

Sharing

The tray had 9 cakes in and they were shared out between Jamie, Kelly and Tony. Each child had the same number of cakes. How many did they have each?



So, 9 ÷ 3 = 3

Grouping

The apples need putting into bags with 5 apples in each bag. Julie has 15 apples. How many bags will she need?

So, 15 ÷ 5 = 3

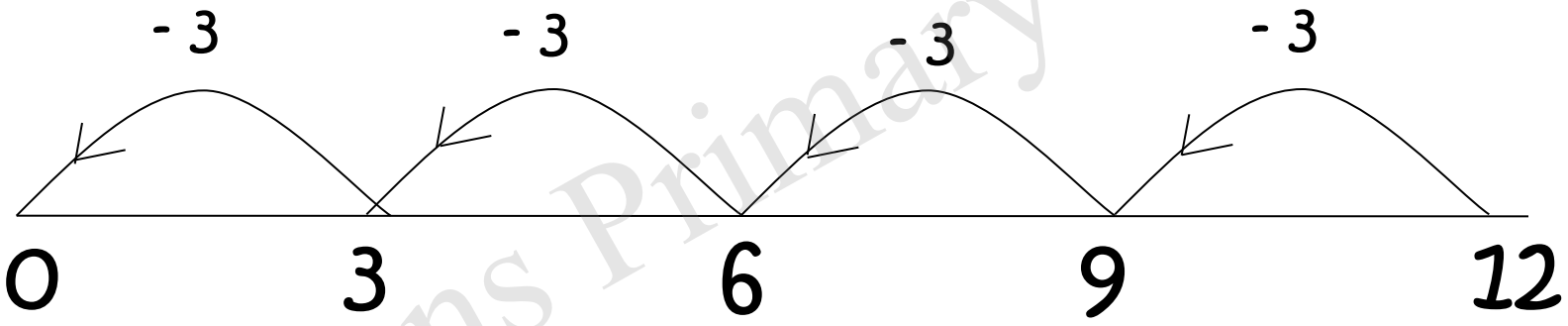
Stage 2: Arrays

First group of 3

$12 \div 3 = 4$

Stage 3: Repeated subtraction

12 ÷ 3 = 4

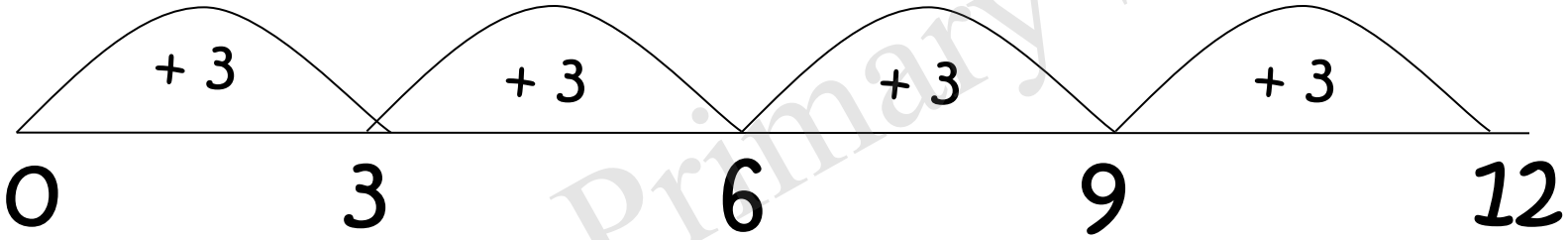


12 - 3 - 3 - 3 - 3 = 4 groups of 3.

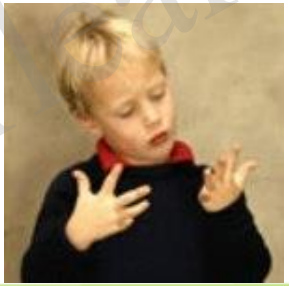
Counting in steps

12 ÷ 3 = 4

Add the jumps



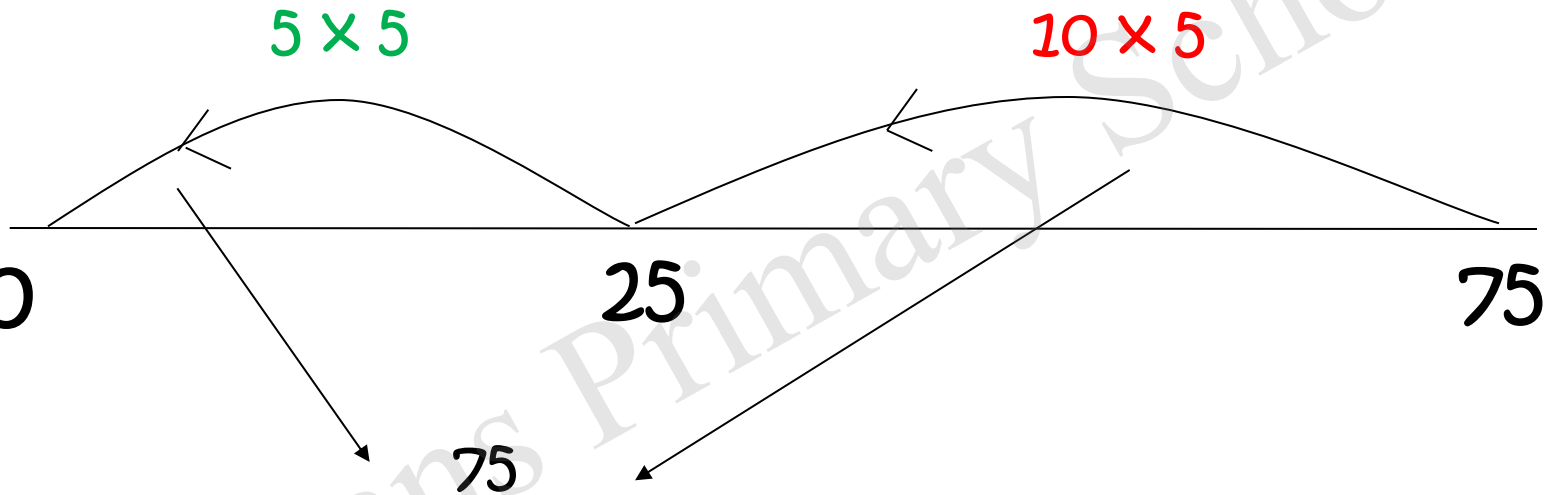
Fingers



“3 “6 “9 “12

Step 4: Chunking

75 ÷ 5



75
- 50 (10 X 5)
25
- 25 (5 X 5)
0

75 ÷ 5 = 15

Need to know tables!

BBC News Video Link

Children need to see that as the numbers get larger, large chunk subtraction is the more efficient method. Multiples of the divisor (large chunks) are taken away. Multiplication facts are needed to see the size of the 'chunk'.

What facts do I know about the 7 times-table?

100 ÷ 7 = 14 r 2

100

- 70 (10 x 7)

30

- 28 (4 x 7)

2

518 ÷ 7 = 74

518

- 350 (50 x 7)

168

- 140 (20 x 7)

28

- 28 (4 x 7)

0

Fact Box

1 x 7 = 7

2 x 7 = 14

5 x 7 = 35

10 x 7 = 70

20 X 7 = 140

50 x 7 = 350

100 x 7 = 700

Stage 5: Short Division

For example: 84 ÷ 7 can be partitioned into (70 + 14) ÷ 7
This can now be calculated: 70 ÷ 7 = 10
14 ÷ 7 = 2 Therefore 84 ÷ 7 = 12

Another example: 104 ÷ 8 = (80 + 24) ÷ 8
80 ÷ 8 = 10
24 ÷ 8 = 3 Therefore 104 ÷ 8 = 13

Remainders may also sometimes need to be recorded:
97 ÷ 6 = (60 + 37) ÷ 6
90 ÷ 6 = 10
37 ÷ 6 = 6 r1 Therefore 97 ÷ 6 = 16 r1

Stage 5: Short Division

The short division method can be recorded like this:

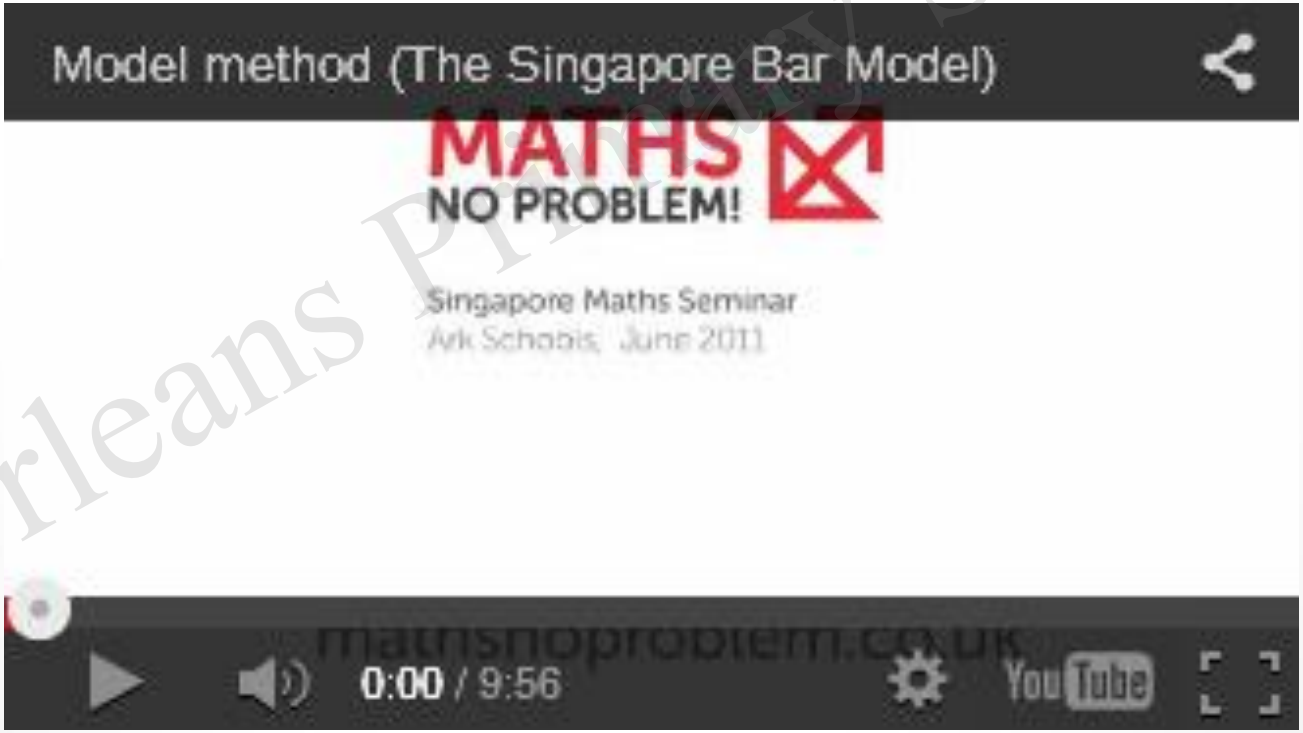
$$\begin{array}{r} 10 + 6 \text{ r1} \\ 6 \overline{)\ 60 + 37} \end{array} = 16 \text{ r1}$$

$$\begin{array}{r} 1\,6 \text{ r1} \\ 6 \overline{)\ 9^{3}7} \end{array}$$

Problem Solving

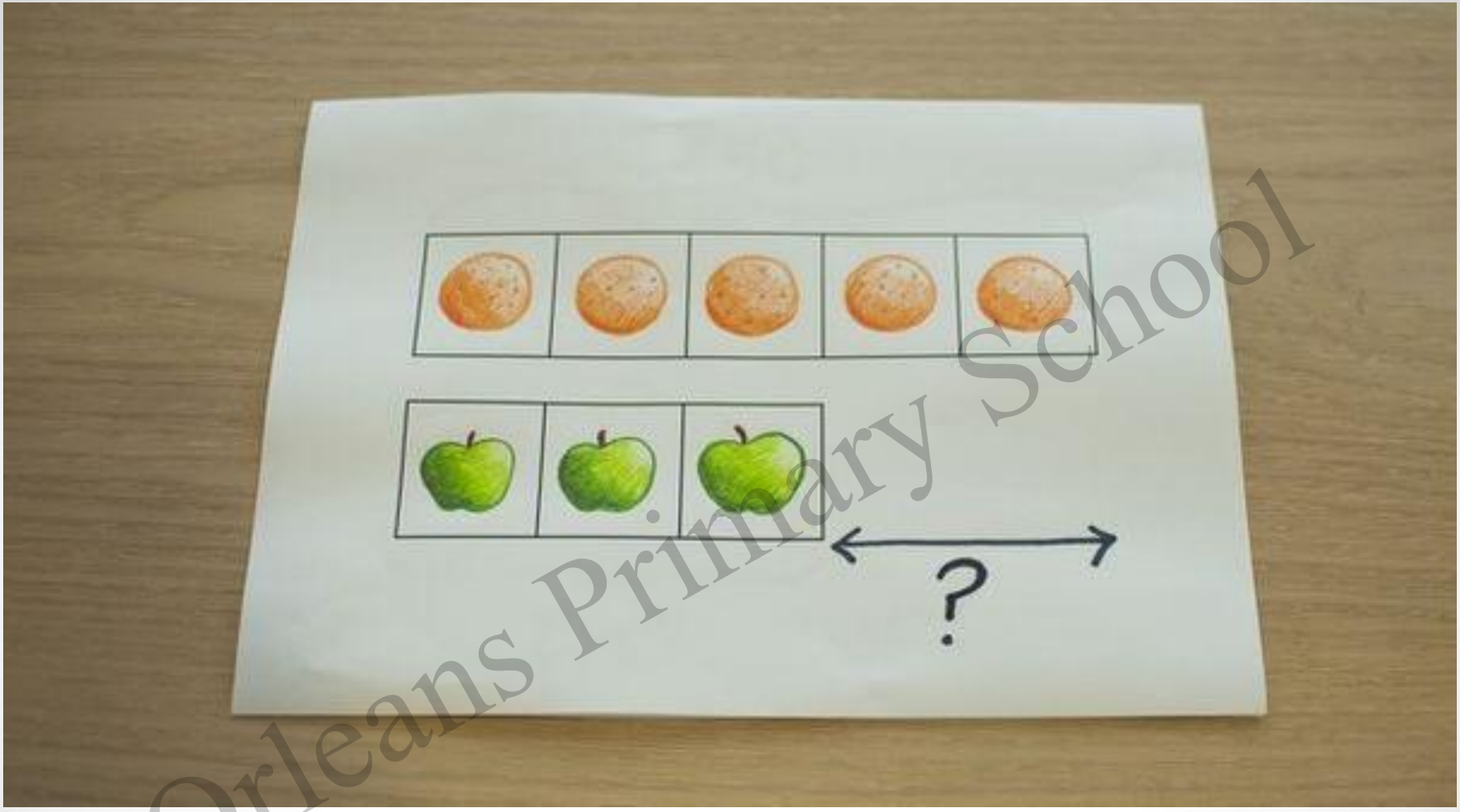
Bar Model

'A picture is worth a thousand words'

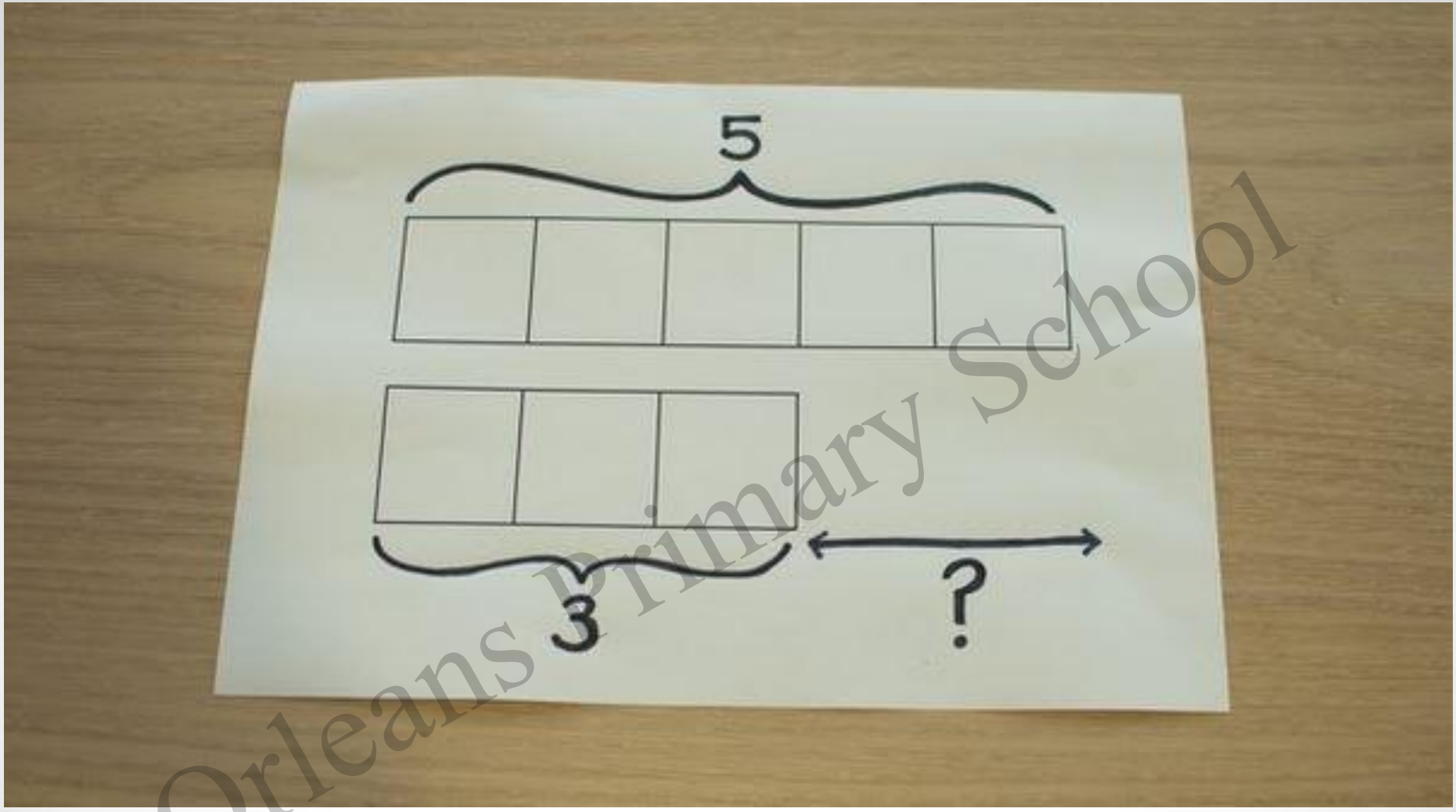


Imagine you have five oranges and three apples.
How many more oranges than apples?

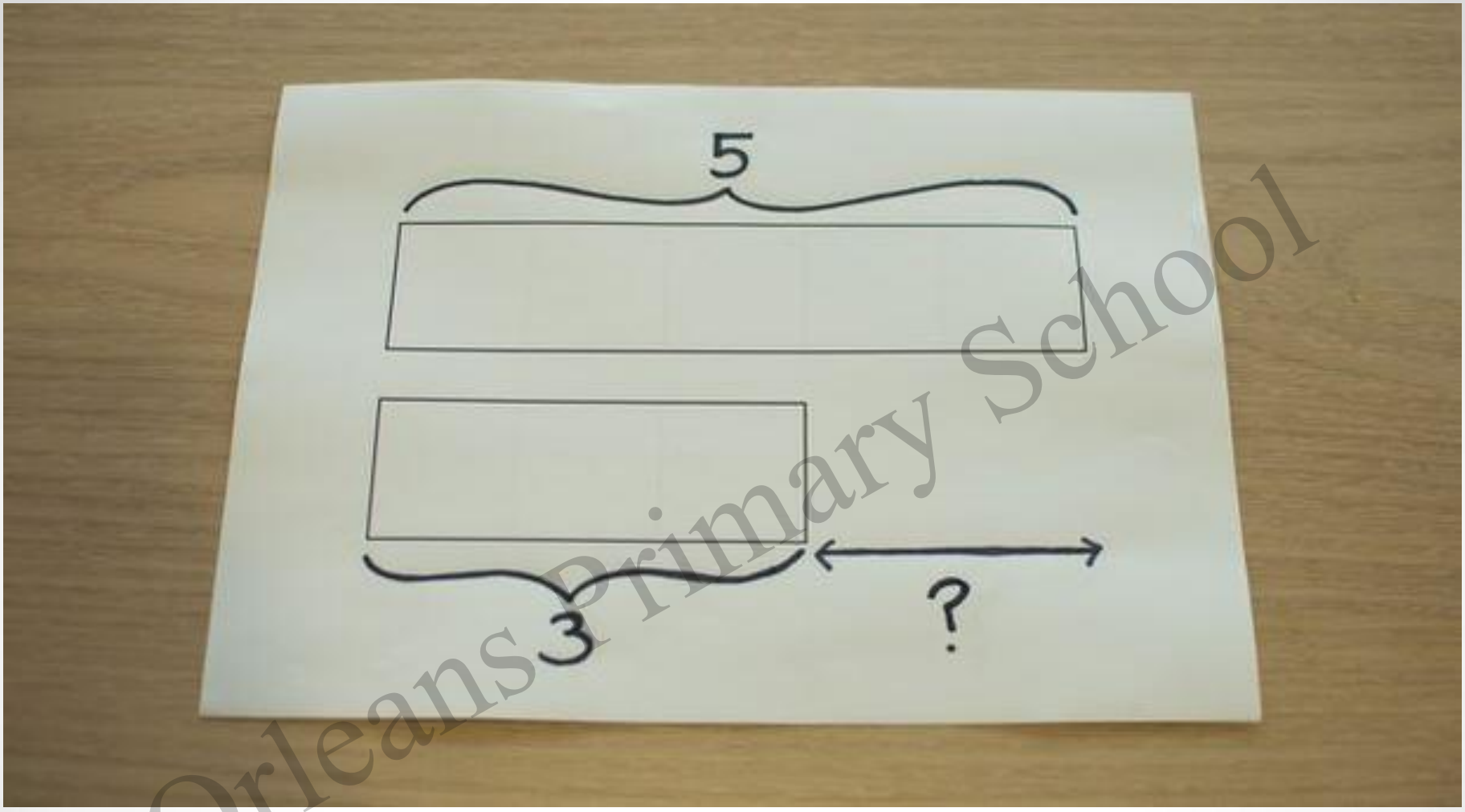
At first children model the problem with physical objects they can move around: like these cut-out pictures.



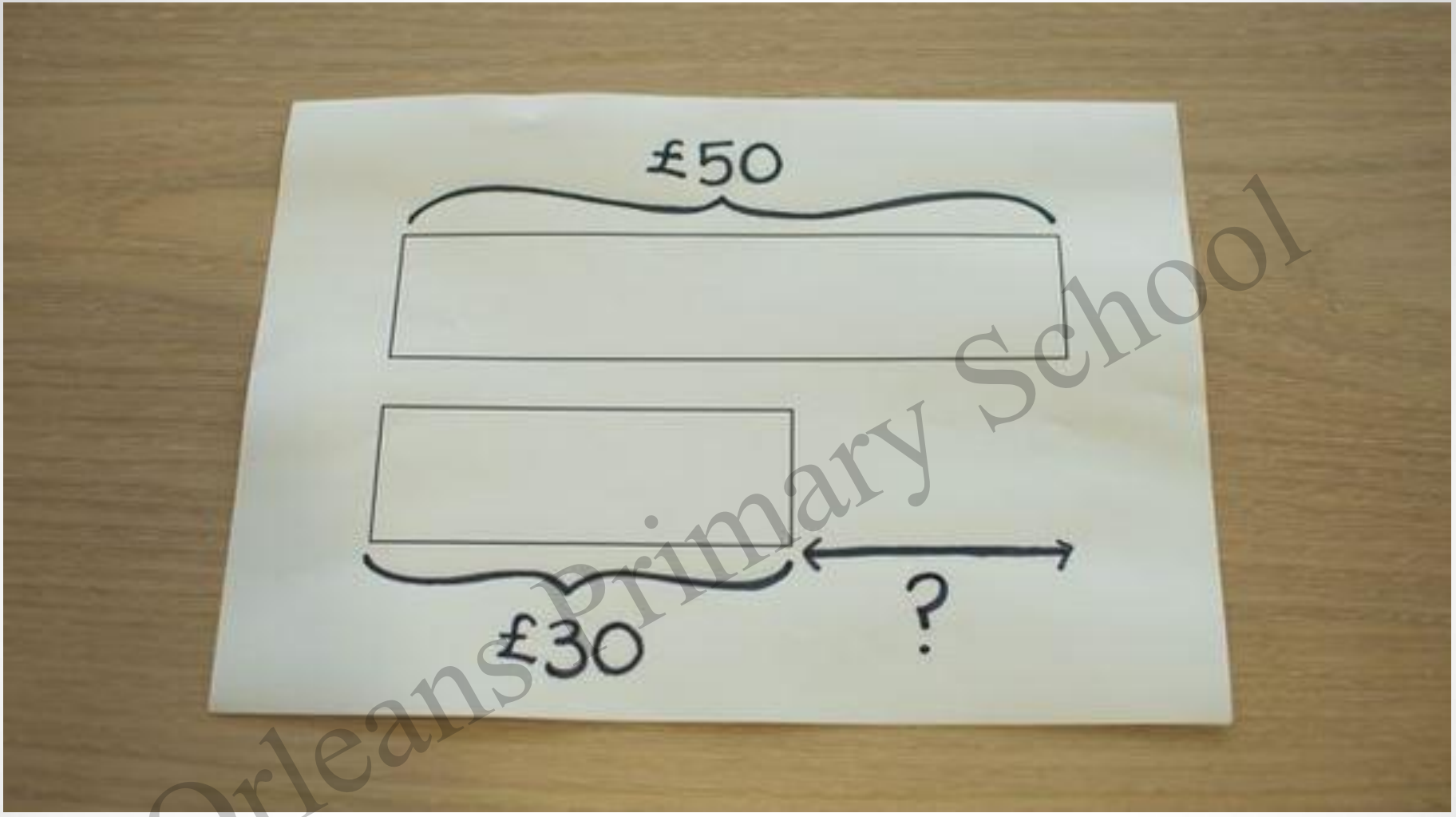
After a few months they start to draw pictures of the problem to help them think about it.



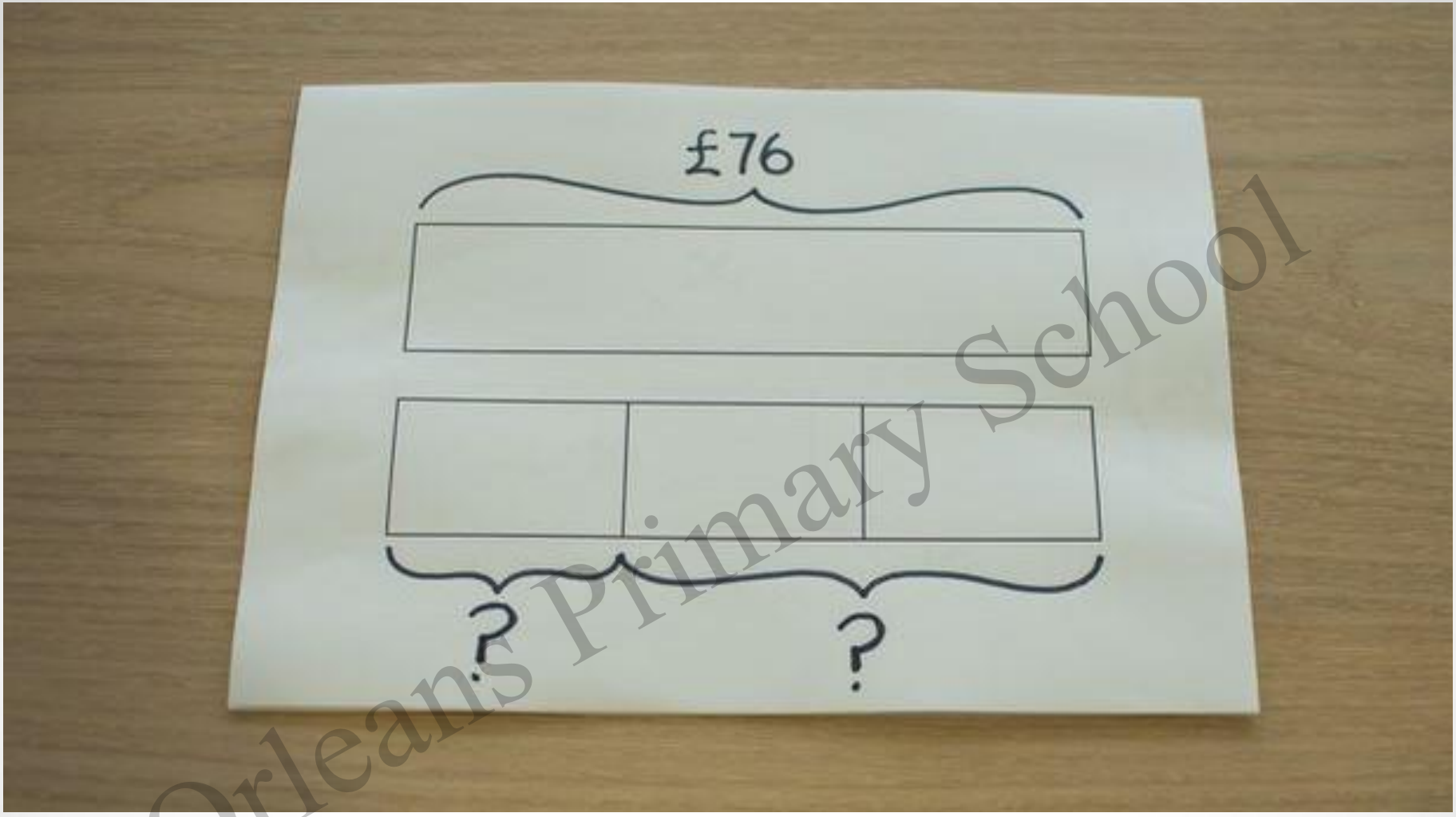
Over time children drop the pictures and just draw boxes. Then they start adding numbers as labels.



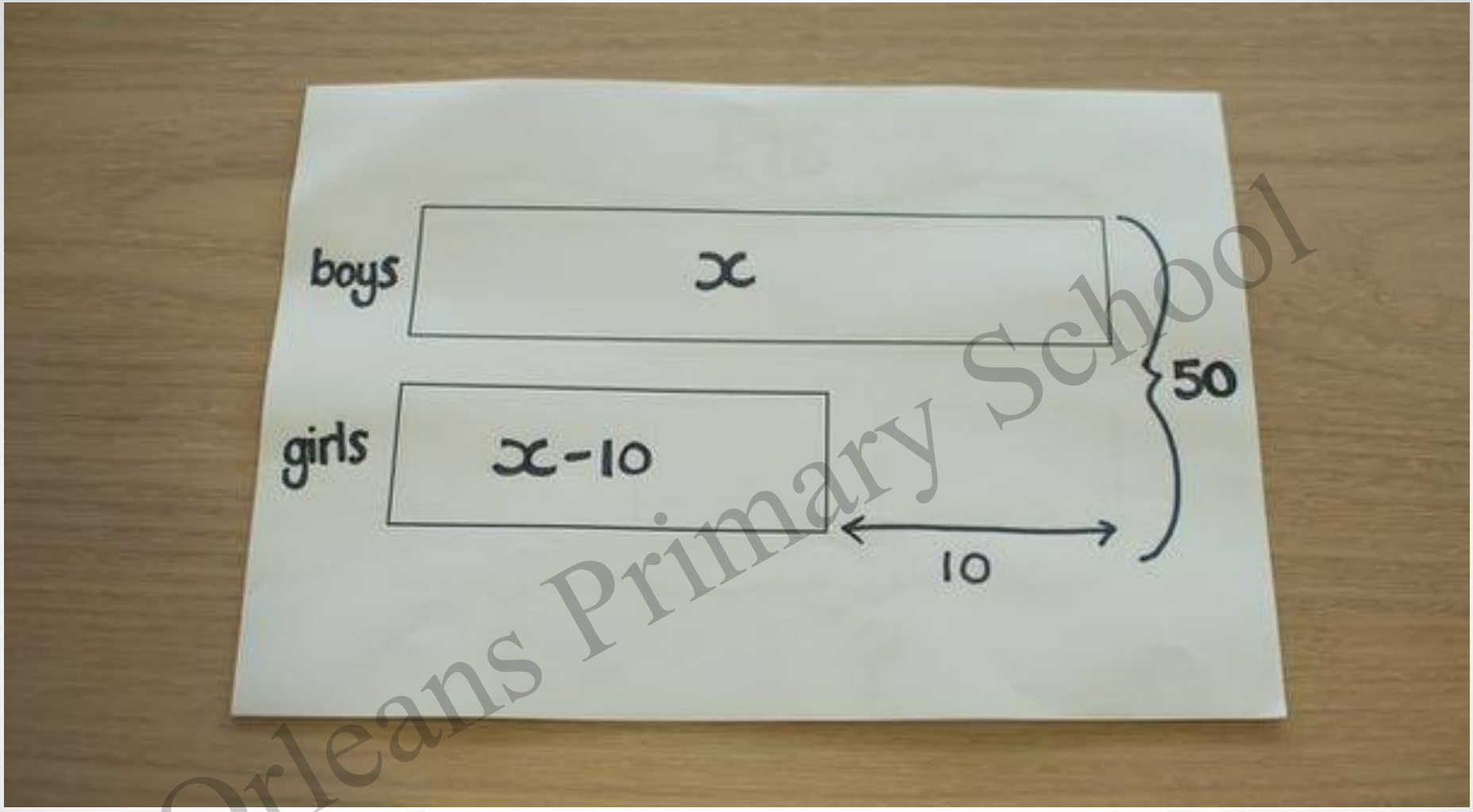
Once children are confident with the meaning of the number symbol they no longer need to draw all the boxes. However they know they can always draw the boxes in again if they need to convince themselves.



How much change if you pay for a £30 shirt with a £50 note? The model can be used to help visualise almost any maths problem.



Three people want to split a restaurant bill of £76. How much for a couple who want to pay together? The model helps break the problem down. First divide £76 by 3. Then times the answer by 2.



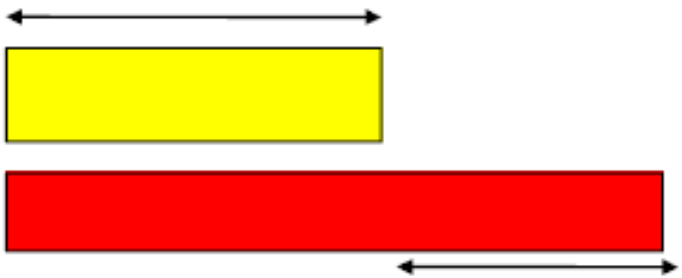
In a year group there are 50 children. There are 10 fewer girls than boys. How many boys? The model can help visualise the unknown quantity. You can see that x + x - 10 = 50. If you add the 10 you get x + x = 60. So x = 30.

Benefits to Bar Models

- Help focus students on *comprehension* of the problem's *situation,* rather than just finding numbers to crunch or just looking for an isolated "key" word or phrase.

- Shows explicitly the problem structure along with the known and unknown quantities

- Visual tool to help students determine the operation needed to solve



Problems to Solve

Tom has a bag of 64 marbles, his friend gives him 28 more, how many does he have now?

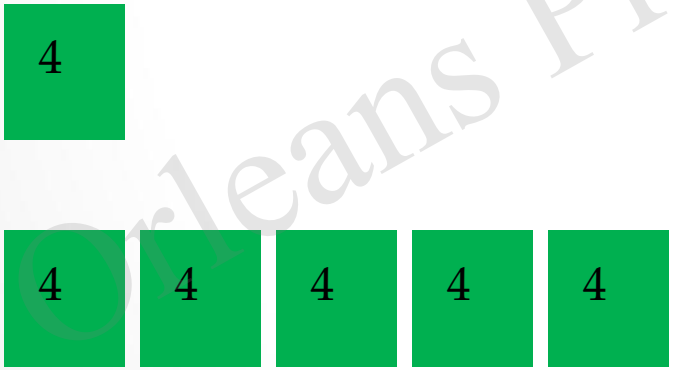
Kelsey was running a 26 mile marathon, after 18 miles she felt very tired. How many more miles did she have to run?

Carly bought an apple for 17p and a banana for 26p, how much has she spent?

Ali had £10, he bought a DVD for £6.70 and a CD for £2. 90, how much money did he have left?

Peter has 4 books

Harry has five times as many books as Peter. How many books has Harry?



Multiplication

- Henry ate 10 meatballs at the Christmas party. Shane ate 3 times as many meatballs as Harry . How many meatballs did they eat altogether?

- Helen has 9 times as many football cards as Sam. Together they have 150 cards. How many more cards does Helen have than Sam?

- The sum of 2 numbers is 60. One number is 9 times as big as the other. What is the bigger number?

- The sum of 2 numbers is 64. One number is 7 times as big as the other. What is the smaller number?

Sam had 5 times as many marbles as Tom. If Sam gives 26 marbles to Tom, the two friends will have exactly the same amount.

How many marbles do they have altogether?

Division

- 108 Year 3 children are going on a field trip to the art museum. Each bus must carry 12 children. How many buses are needed?

- Mr Smith had a piece of wood that measured 36 cm. He cut it into 6 equal pieces. How long was each piece?

Solving Proportional Problems

- Peter has ten sweets he eats half of them how many does he have left?
- Ali has 30 sweets, she eats 1/3 of them, how many does she have left?
- Stacey has 30 sweets, she eats 2/3 of them, how many does she have left?
- A dress costs £32, it is reduced in price by 50%, how much does it cost know?

- $^{3}/_{8}$ of the sweets in the tin were chocolates. $^{1}/_{4}$ were toffees. The rest were strawberry creams. There were 36 strawberry creams. How many sweets were in the tin?

- Annie answered $^{4}/_{5}$ of the questions on the test. She answered 32. How many questions were on the test?

- A pair of jeans was reduced in a sale by 25%. They now cost £48. What was the original price?

- A Super Mario Game costs £45, it is reduced in price by 25%, how much does it cost now?
- A computer game was reduced in a sale by 20%, it now costs £40, what was the original price?
- A computer game was reduced in a sale by 40%, it now costs £60, what was the original cost?
- Laura had £240. She spent 5/8 of it. How much money did she have left?

Ratio

Tim and Sally share marbles in the ratio of 2:3
If Sally has 36 marbles, how many are there altogether?

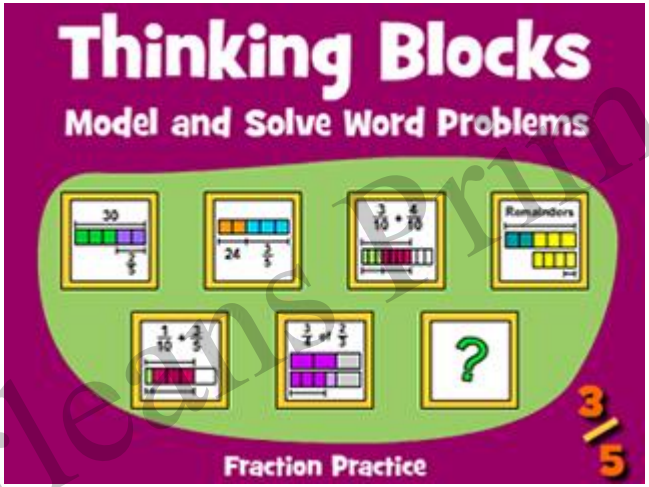
A herbal skin remedy uses honey and yoghurt in the ratio 3 : 4. How much honey is needed to mix with 120 g of yoghurt?

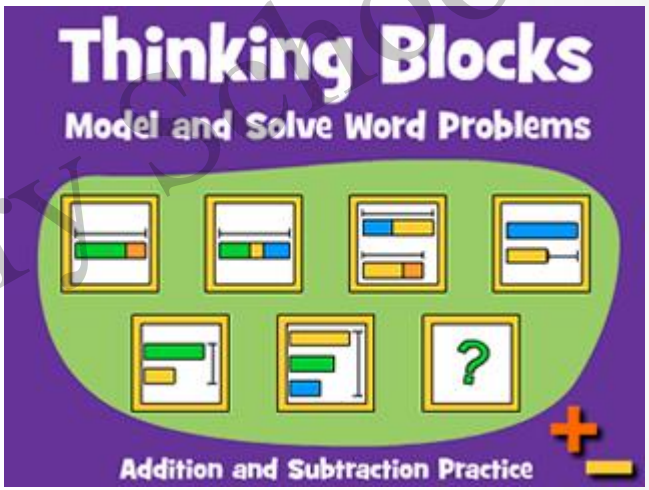
A health bar sells desserts with chopped apricot and yoghurt in the ratio 2 : 5. How much chopped apricot will be mixed with 150 g of yoghurt?

At peak times my mobile phone costs 1 times as much as it does off-peak. A peak call costs 90p. What would it have cost off-peak?

Bar Method Modelling

1. www.mathplayground.com
   - Thinking Blocks





2. www.thesingaporemaths.com